Torsion
Torque

- Def: Moment along an axle or shaft
- Equilibrium: Torque out = Torque in
Plane sections remain planar in torsion.

- Before deformation (a):
  - Circles remain circular.
  - Radial lines remain straight.

- After deformation:
  - Longitudinal lines become twisted.

Diagram shows a cylinder before and after deformation, illustrating the changes in circular and straight lines due to torsion.
Side view of a twisted shaft

BEFORE

AFTER TORSION APPLIED
The angle of twist $\phi(x)$ increases as $x$ increases.
Shear stress is linear with radius
Fundamental Equations

\[ \tau = G \gamma \]

\[ \tau = \frac{Tr}{J} \]

\[ \theta = \frac{Tl}{JG} \]
Low Stress at center of shaft is why many shafts are hollow

Shear stress varies linearly along each radial line of the cross section.
The gears attached to the fixed-end steel shaft are subjected to the torques shown in Fig. 10–18a. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm, determine the displacement of the tooth \( P \) on gear \( A \). The shaft turns freely within the bearing at \( B \).

**SOLUTION**

**Internal Torque.** By inspection, the torques in segments \( AC, CD, \) and \( DE \) are different yet constant throughout each segment. Free-body diagrams of appropriate segments of the shaft along with the calculated internal torques are shown in Fig. 10–18b. Using the right-hand rule and the established sign convention that positive torque is directed away from the sectioned end of the shaft, we have

\[
T_{AC} = +150 \text{ N} \cdot \text{m} \quad T_{CD} = -130 \text{ N} \cdot \text{m} \quad T_{DE} = -170 \text{ N} \cdot \text{m}
\]

These results are also shown on the torque diagram, Fig. 10–18c.
**Angle of Twist.** The polar moment of inertia for the shaft is

\[ J = \frac{\pi}{2} (0.007 \text{ m})^4 = 3.771 \times 10^{-9} \text{ m}^4 \]

Applying Eq. 10–16 to each segment and adding the results algebraically, we have

\[
\phi_A = \sum \frac{TL}{JG} = \frac{(+150 \text{ N} \cdot \text{m})(0.4 \text{ m})}{3.771 \times 10^{-9} \text{ m}^4 [80(10^9) \text{ N/m}^2]} + \frac{(-130 \text{ N} \cdot \text{m})(0.3 \text{ m})}{3.771 \times 10^{-9} \text{ m}^4 [80(10^9) \text{ N/m}^2]} \]
\[+ \frac{(-170 \text{ N} \cdot \text{m})(0.5 \text{ m})}{3.771 \times 10^{-9} \text{ m}^4 [80(10^9) \text{ N/m}^2]} = -0.2121 \text{ rad} \]

Since the answer is negative, by the right-hand rule the thumb is directed toward the end E of the shaft, and therefore gear A will rotate as shown in Fig. 10–18d.

The displacement of tooth P on gear A is

\[ s_P = \phi_A r = (0.2121 \text{ rad})(100 \text{ mm}) = 21.2 \text{ mm} \quad \text{Ans.} \]

**NOTE:** Remember that this analysis is valid only if the shear stress does not exceed the proportional limit of the material.
END