MAT 260 Fall 2004 Dr. Yackel's Section
Induction Theorems

Definition: In **weak induction** one assumes that $P(n)$ is true and proves that $P(n+1)$ is true. (Also, one completes the base case, of course.)

Definition: In **strong induction** one assumes that $P(i)$ is true for all values of i greater than or equal to the value in the base case and less than or equal to n and proves that $P(n+1)$ is true. (Also, one completes the base case, of course.)

Prove the following statements using either strong or weak induction.

(1) Pascal's Formula: Let n and r be positive integers and suppose that $r \leq n$. Then
$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}.$$
(2) Binomial Theorem: Let n be a nonnegative integer. Then
$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r.$$
(3) Let m be any nonnegative integer. Let $n \geq 0$ be an integer. Then
$$\binom{m}{0} + \binom{m+1}{1} + \cdots + \binom{m+n}{m} = \binom{m+n+1}{n}.$$
(4) If $n \geq 4$ is an integer, then $n! > 2^n$.

(5) $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(6) By thinking about the proof without words that $\sum_{i=0}^{n} 2i+1 = (n+1)^2$, find a similar sum whose value is $(n+1)^3$. Now try $(n+1)^4$. Can you write a sum for $(n+1)^k$?

(7) $4|5^n - 1$, for integers $n \geq 0$.

(8) $6|7^n - 1$, for integers $n \geq 0$.

(9) Does $k-1|k^n - 1$ for all integers $n \geq 0$ and for all integers $k \geq 2$?