

1. OBSERVATION, SEQUENCING JOBS BY DUE DATE ORDERING IS A GOOD HEURISTIC, AND CAN PROVIDE A GOOD UPPER BOUND.

i	p_i	d_i	C_i	T_i
1	8	10	8	0
2	2	12	10	0
3	6	14	16	2
4	4	16	20	4
5	5	18	25	7

13 = UPPER BOUND 1, UB1

ALSO CONSIDER THAT JOBS WITH LOW DUE DATES AND SHORT PROCESS TIMES ARE CANDIDATES TO GO FIRST. SO TRY ANOTHER UPPER BOUND BY SORTING JOB BY SUM OF PROCESS TIME + DUE DATE.

i	p_i	d_i	C_i	T_i
2	2	12	2	0
1	8	10	10	0
3	6	14	16	2
4	4	16	20	4
5	5	18	25	7

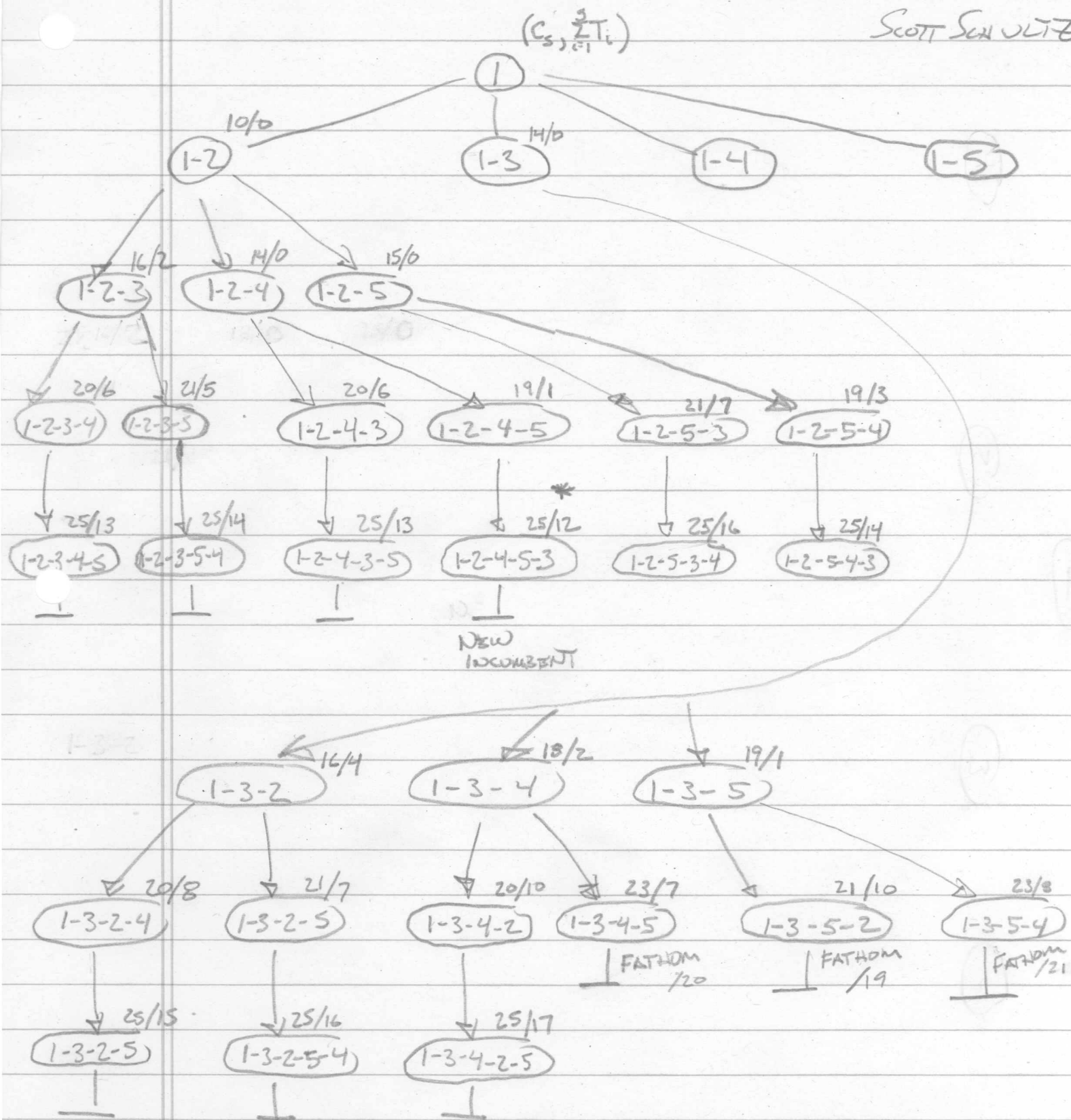
13 = UPPER BOUND 2, UB2

BOTH UPPER BOUNDS = 13.

- * OBSERVATION, JOB 1 OR 2 CANNOT BE LAST
 $i=1$ $25-10, T_1=15$ $i=2$ $25-12=13, T_2=13$ (WHEN LAST)

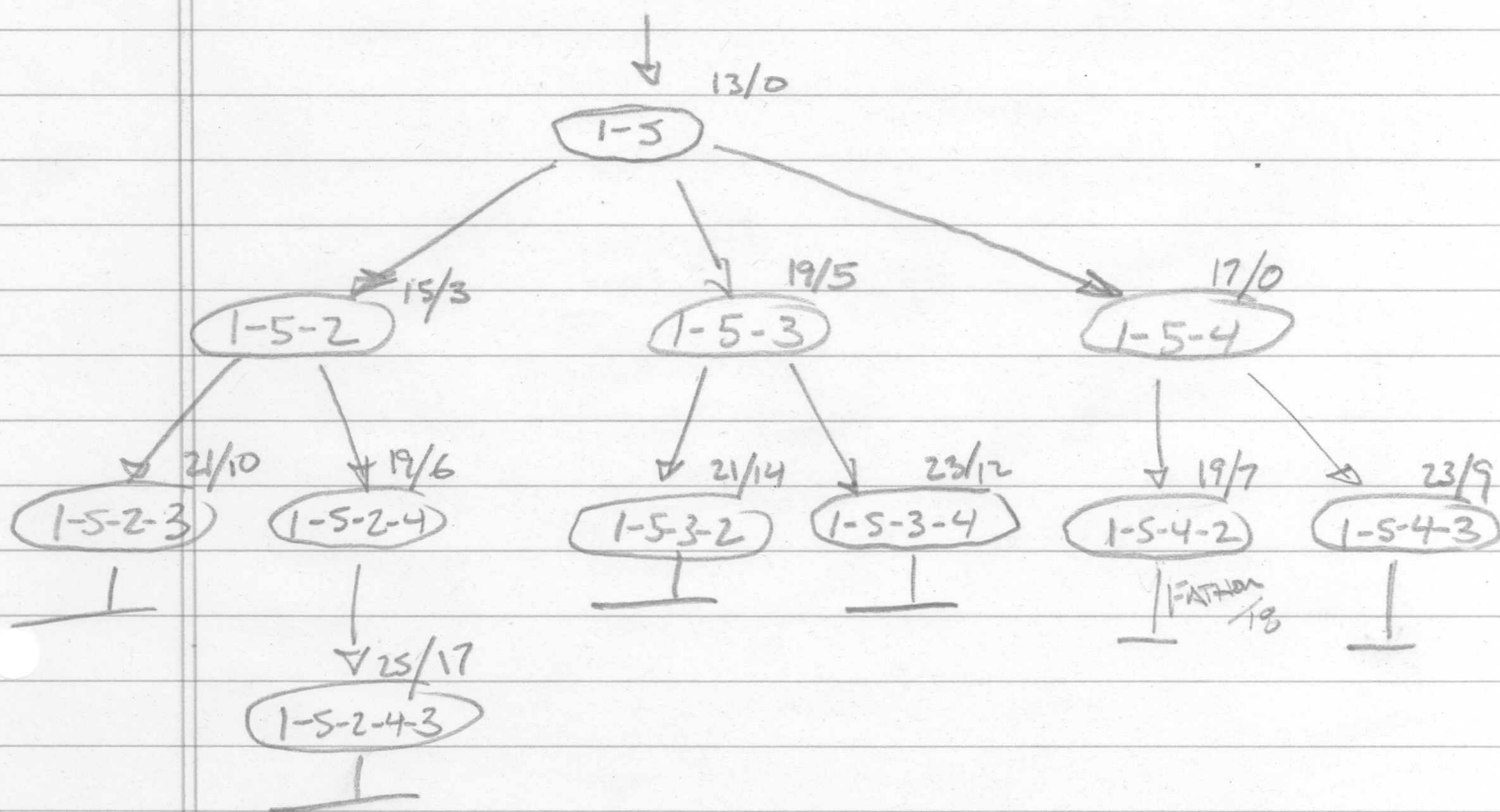
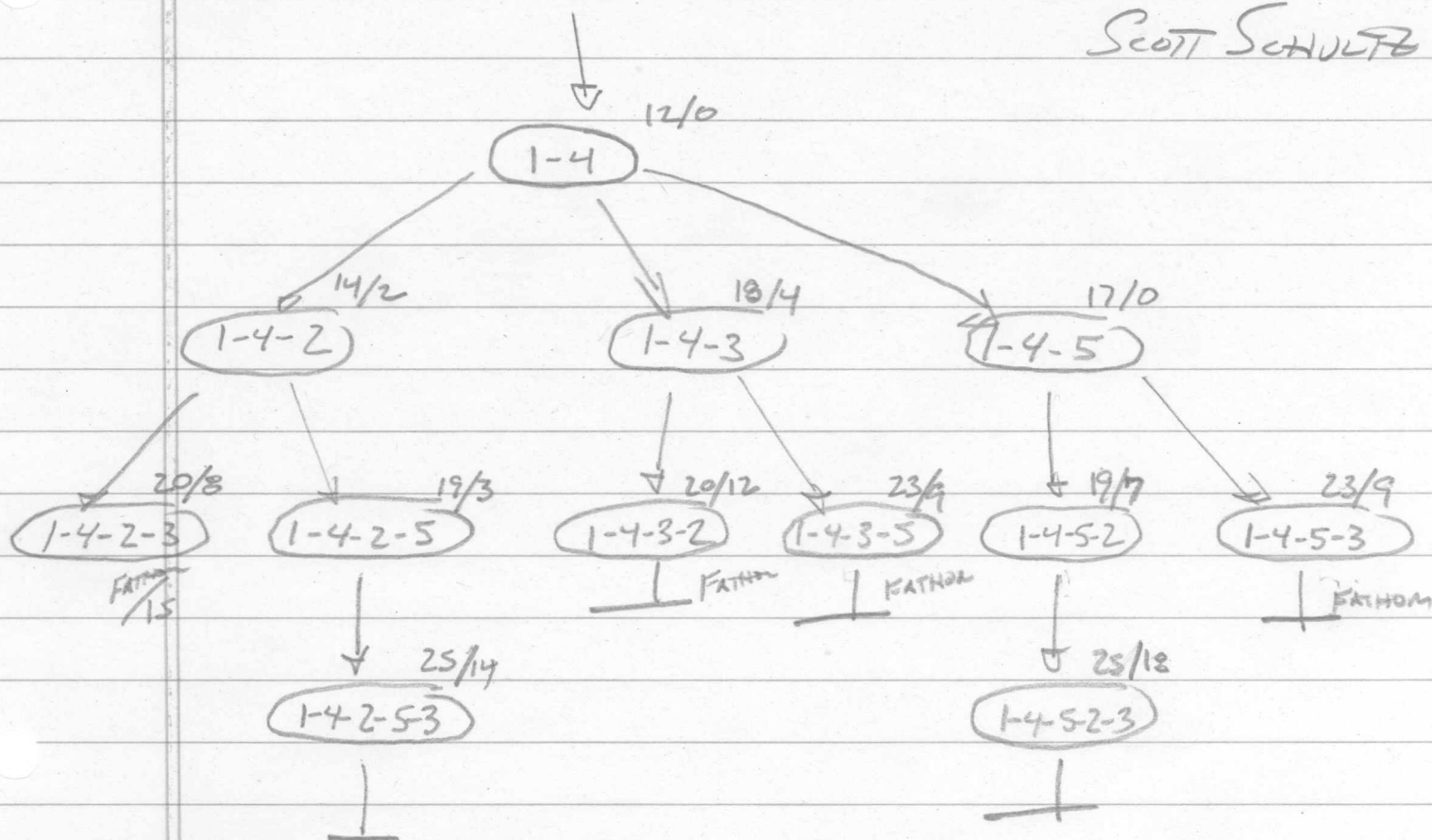
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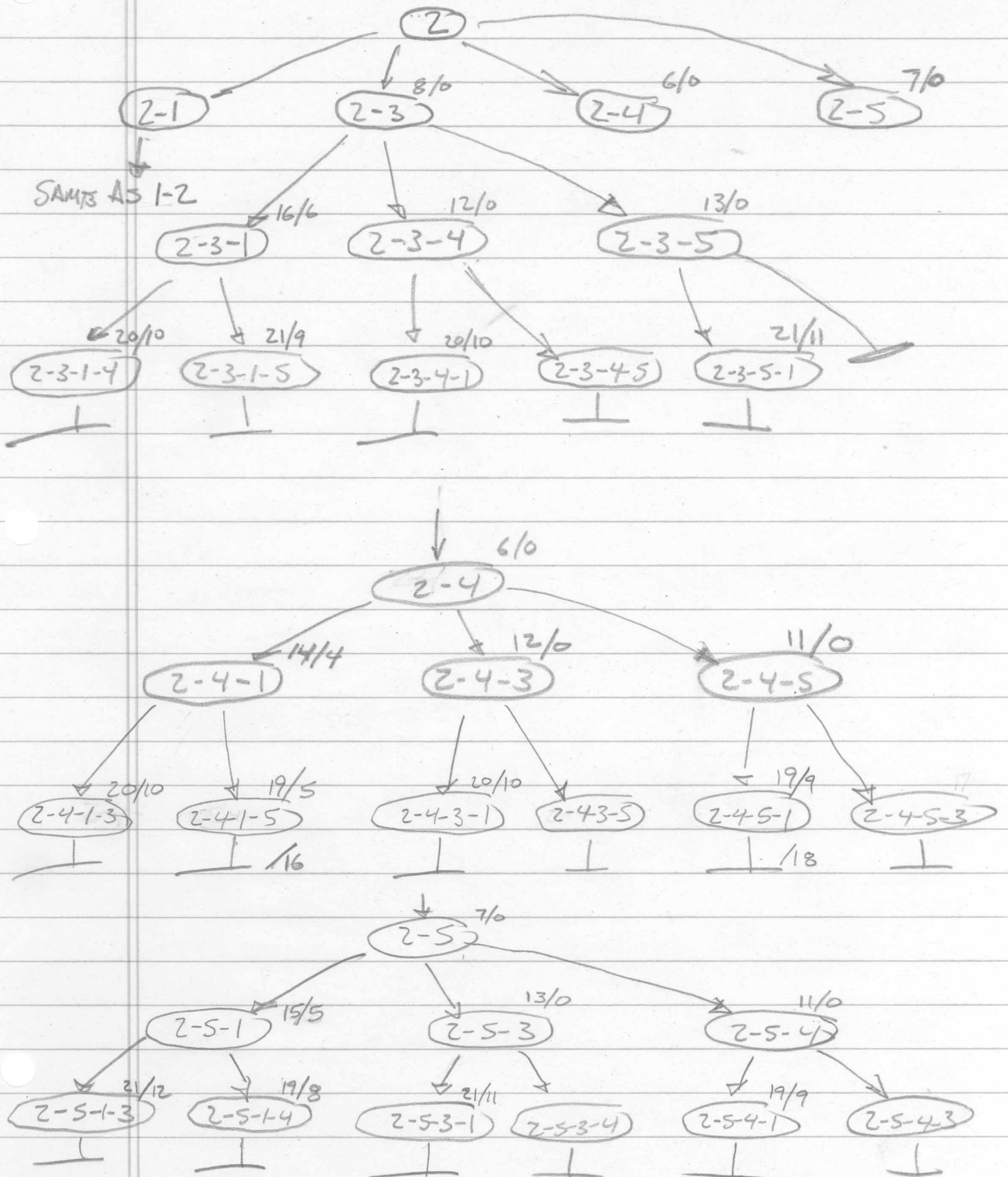
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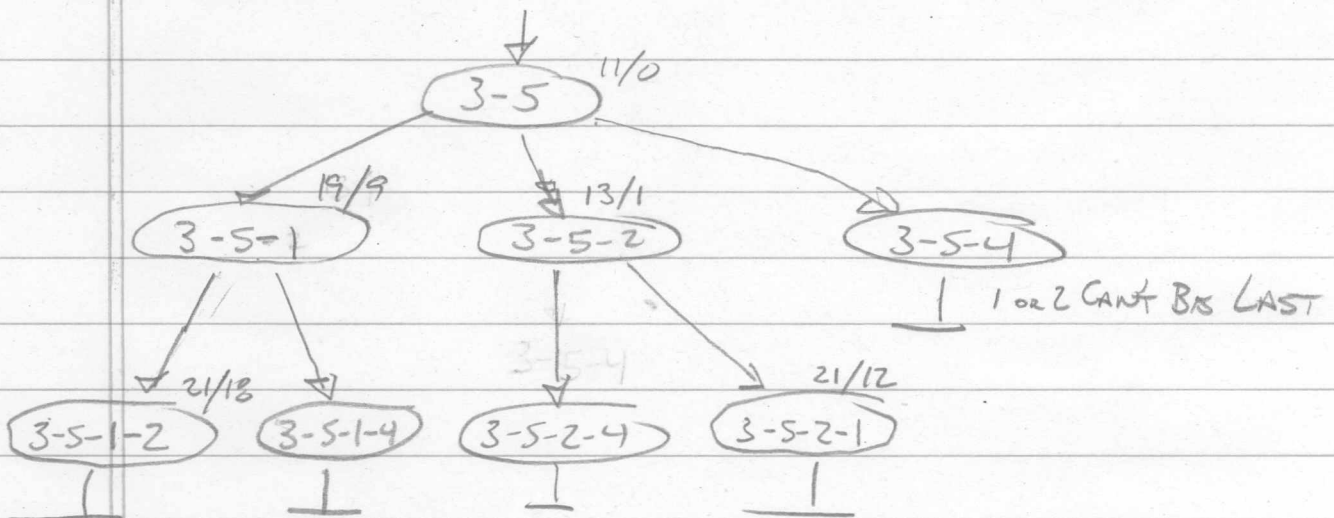
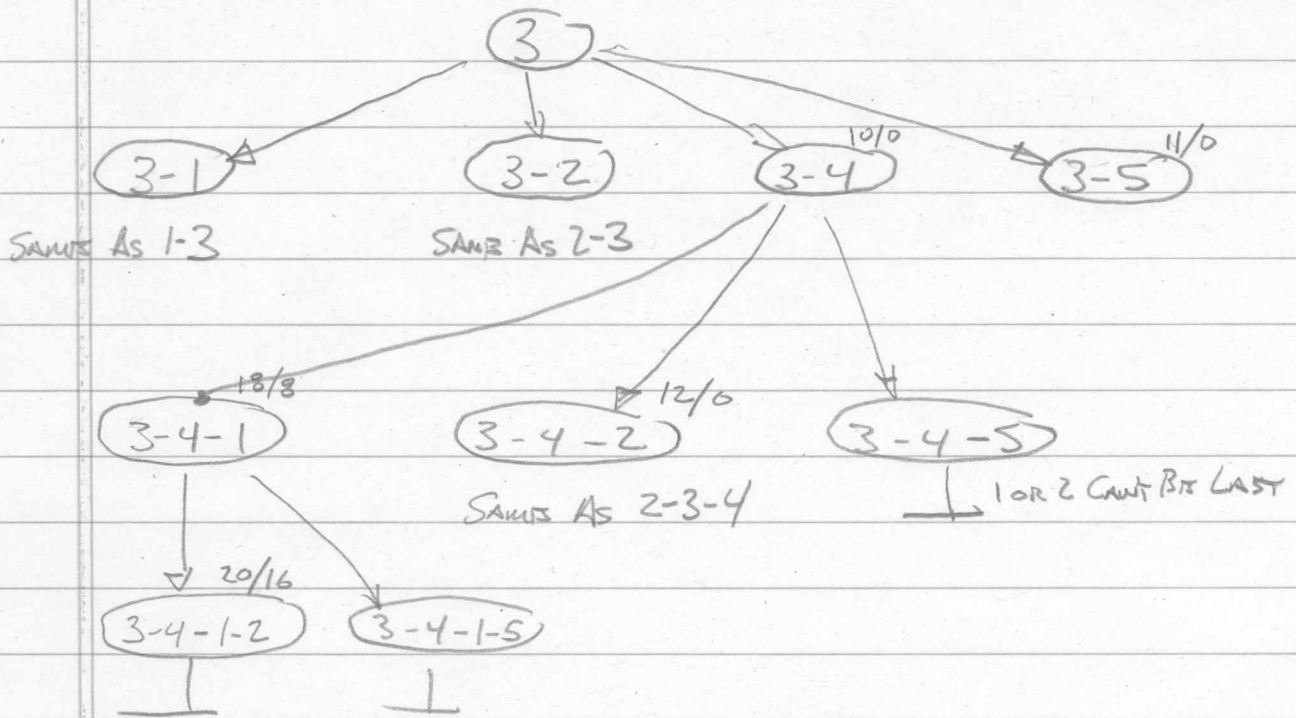
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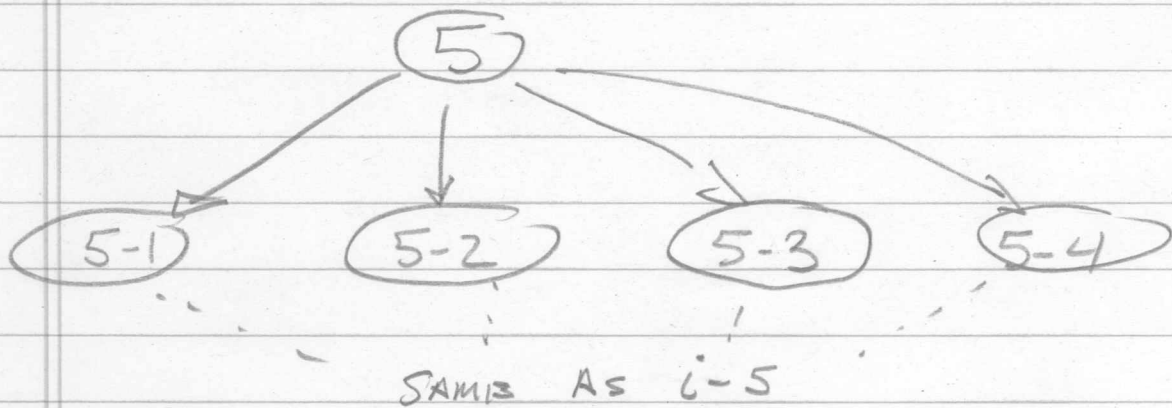
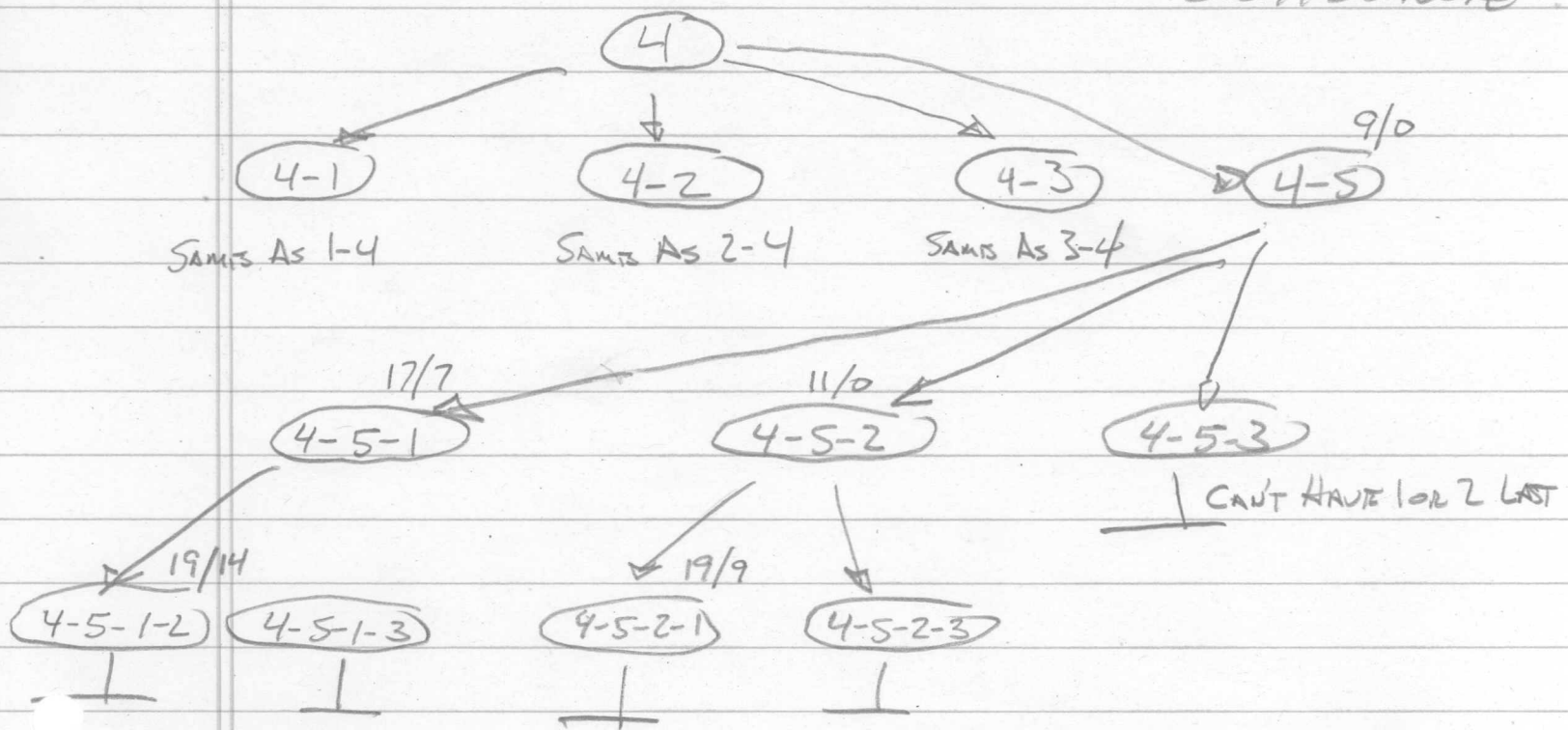
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OPTIMAL SEQUENCE

1-2-4-5-3

$\Sigma T = 12$

OR

2-1-4-5-3

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2. MAX $Z = 14X_1 + 18X_2$

$$-X_1 + 3X_2 + S_1 = 6$$

$$7X_1 + X_2 + S_2 = 35$$

$$X_1, X_2 \geq 0 \quad X_1, X_2 \text{ are integers}$$

OPTIMAL TABULAR FROM LP RELAXATION

Z	X_1	X_2	S_1	S_2	rhs	
1	0	0	$56/11$	$30/11$	126	Row 0
0	0	1	$7/22$	$1/22$	$7/2$	Row 1
0	1	0	$-1/22$	$3/22$	$9/2$	Row 2

CHOOSE CUT ON CONSTRAINT 1/ROW 1:

$$X_2 + \frac{7}{22}S_1 + \frac{1}{22}S_2 = \frac{7}{2}$$

REWRITES TO LOWEST INT + FRAC. $\rightarrow X_2 + 0S_1 + \frac{7}{22}S_1 + 0S_2 + \frac{1}{22}S_2 = 3 + \frac{1}{2}$

MOVES FRACTIONS TO RIGHT SIDE $\rightarrow X_2 - 3 = \frac{1}{2} - \frac{7}{22}S_1 - \frac{1}{22}S_2$

NEW CUTTING CONSTRAINT: $\frac{1}{2} - \frac{7}{22}S_1 - \frac{1}{22}S_2 \leq 0$

OR

$$-\frac{7}{22}S_1 - \frac{1}{22}S_2 + S_3 = -\frac{1}{2}$$

$\rightarrow$

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2 cont.)

NEW TABLEAU WITH ADDED CONSTRAINT:

	<u>Z</u>	<u>X₁</u>	<u>X₂</u>	<u>S₁</u>	<u>S₂</u>	<u>S₃</u>	<u>rhs</u>
Row 0	1	0	0	$\frac{56}{11}$	$\frac{30}{11}$	0	126
1	0	0	1	$\frac{7}{22}$	$\frac{1}{22}$	0	$\frac{7}{2}$
2	0	1	0	$-\frac{1}{22}$	$\frac{3}{22}$	0	$\frac{9}{2}$
3	0	0	0	$-\frac{7}{22}$	$-\frac{1}{22}$	1	$-\frac{1}{2}$

LEAVING BASIC VARIABLE - S₃ because negative r.h.s.

MIN RATIO RULE - choose min from absolute value of $\frac{\text{Row 0 Coeff}}{\text{Leaving Basic Var Coeff}}$

entering non-basic $\begin{cases} S_1 - \frac{56/11}{7/22} = 16 \quad \text{choose } S_1 \\ S_2 - \frac{30/11}{1/22} = 60 \end{cases}$

MANIPULATE TABLEAU ABOUT S₀: S₁ enters + S₃ leaves basis

	<u>Z</u>	<u>X₁</u>	<u>X₂</u>	<u>S₁</u>	<u>S₂</u>	<u>S₃</u>	<u>rhs</u>	
Row 0	1	0	0	0	2	16	118	NR0 = OR0 - $\frac{56}{11}$ NR3 (STEP 4)
1	0	0	1	0	0	1	3	NR1 = OR1 - $\frac{7}{22}$ NR3 (STEP 3)
2	0	1	0	0	$\frac{1}{7}$	$-\frac{1}{7}$	$\frac{32}{7}$	NR2 = OR2 + $\frac{1}{22}$ NR3 (STEP 2)
3	0	0	0	1	$\frac{1}{7}$	$-\frac{22}{7}$	$\frac{11}{7}$	NR3 = $-\frac{22}{7}$ OR3 (STEP 1)

NEW CORNER POINT $X_1 = \frac{32}{7}$ $X_2 = 3$ $Z = 118$

NOT INTEGER, SO ADD ANOTHER CUT BASED ON ROW 2



2 cont. CONSTRAINT 2/ROW 2:

$$x_1 + \frac{1}{7}S_2 - \frac{1}{7}S_3 = \frac{32}{7}$$

REWRITES TO LOWEST INT + FRACT. $\Rightarrow x_1 + 0S_2 + \frac{1}{7}S_2 - 1S_3 + \frac{6}{7}S_3 = 4 + \frac{4}{7}$

MOVES FRACTIONS TO RIGHT SIDE $\Rightarrow x_1 - 1S_3 - 4 = -\frac{1}{7}S_2 - \frac{6}{7}S_3 + \frac{4}{7}$

NEW CUTTING CONSTRAINT: $\frac{4}{7} - \frac{1}{7}S_2 - \frac{6}{7}S_3 \leq 0$

OR

$$-\frac{1}{7}S_2 - \frac{6}{7}S_3 + S_4 = -\frac{4}{7}$$

NEW TABLEAU WITH ADDED CONSTRAINT:

	$\bar{z}$	x_1	x_2	s_1	s_2	s_3	s_4	rhs
Row 0		1	0	0	0	2	16	0
1		0	0	1	0	0	1	0
2		0	1	0	0	$\frac{1}{7}$	$-\frac{1}{7}$	$\frac{32}{7}$
3		0	0	0	1	$\frac{1}{7}$	$-\frac{22}{7}$	$\frac{11}{7}$
4		0	0	0	$-\frac{1}{7}$	$\frac{6}{7}$	1	$-\frac{4}{7}$

 s_4 - leaves (negative r.h.s.)

min ratio rule $\begin{cases} s_2 - \frac{2}{17} \\ s_3 - \frac{16}{17} \end{cases} \leftarrow s_2 \text{ enters}$

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Z cont.) PERFORM PIVOT (S_4 -leaves, S_2 -enters)

	<u>Z</u>	<u>x_1</u>	<u>x_2</u>	<u>s_1</u>	<u>s_2</u>	<u>s_3</u>	<u>s_4</u>	<u>rhs</u>	
Row 0	1	0	0	0	0	100	14	110	$NR0 = OR0 + 2$
1	0	0	1	0	0	1	0	3	$NR1 = OR1$
2	0	1	0	0	0	$\frac{5}{7}$	1	4	$NR2 = OR2 + OR1$
3	0	0	0	1	0	$-\frac{16}{7}$	1	1	$NR3 = OR3 + OR1$
4	0	0	0	0	1	-42	-7	4	$NR4 = OR4 + (-7)$

 x_1, x_2 ARE INTEGER $\therefore$ OPTIMAL FOR ORIGINAL PROBLEM

$$x_2 = 3$$

$$x_1 = 4$$

$$Z = 110$$