LEO COCO: MAX $20X_1 + 10X_2$ ①

st $X_1 - X_2 \le 1$ ②

$3X_1 + X_2 \le 7$ ③

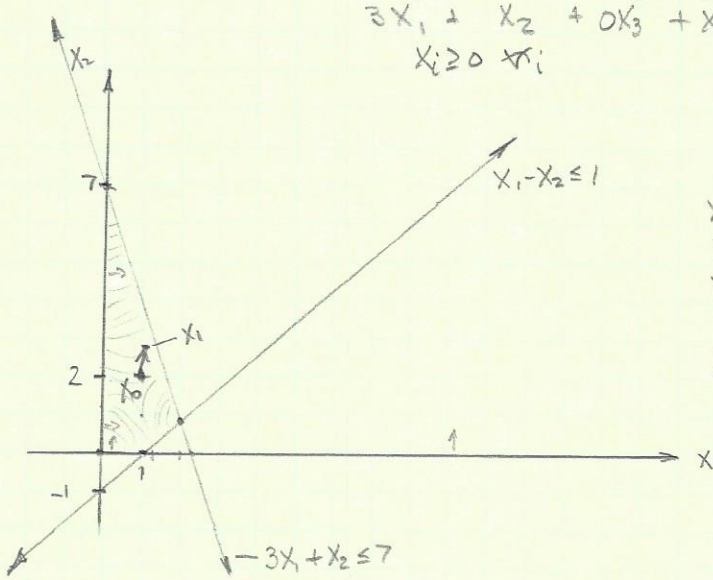
$X_1 \ge 0, X_2 \ge 0$

$-(\text{MIN})\ -20X_1 - 10X_2$ ④

$X_1 - X_2 + X_3 + 0X_4 = 1$ ⑤

$3X_1 + X_2 + 0X_3 + X_4 = 7$ ⑥

$X_i \ge 0\ \forall i$



$X_1 - X_2 \le 1$ (0 -1)(1 0)

$3X_1 + X_2 \le 7$ (0 7)(7/3, 0)

$X_0 = (1, 2)$

$X_1 = (1.2906, 2.7265)$

$X_2 = (0.9642182, 3.9120919)$

$X_3 = (0.4125141, 5.6148544)$

$X_4 = (0.0662357, 6.7019618)$

$X_5 = (0.0314153, 6.8849245)$

$X_6 = (0.0065492, 6.9704585)$

$X_7 = (0.0031151, 6.9885986)$

CHOOSE AN INITIAL POINT IN THE INTERIOR OF THE 2D FEAS. REGION (e.g. (1, 2)

SUB $X_1 = 1, X_2 = 2$ INTO ⑤ & ⑥ ABOVE AND SOLVE FOR X_3, X_4 TO OBTAIN $\bar{X}_0$

$1 - 2 + X_3 = 1 \qquad X_3 = 2$

$3(1) + 2 + X_4 = 7 \qquad X_4 = 2$

$$\bar{x}_0 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \end{bmatrix} \qquad \bar{X}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \qquad \bar{y}_0 = \bar{X}_0^{-1} \bar{x}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \end{bmatrix}$$

$$\bar{y}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

so $y_1 = \frac{x_1}{1} \quad x_1 = y_1$

$y_2 = \frac{x_2}{2} \quad x_2 = 2y_2$

$y_3 = \frac{x_3}{2} \quad x_3 = 2y_3$

$y_4 = \frac{x_4}{2} \quad x_4 = 2y_4$

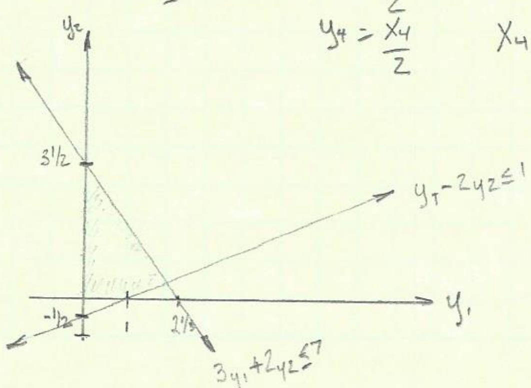
$\Rightarrow$ LEO COCO X-FORMED

$-(\text{MIN})\ -20y_1 - 20y_2$

$y_1 - 2y_2 \le 1$ $(0, -\frac{1}{2})(1, 0)$

$3y_1 + 2y_2 \le 7$ $(0, 7/2)(7/3, 0)$

$y_1 \ge 0, y_2 \ge 0$



$\Rightarrow$ FIND DIRECTION TO MOVE $\bar{d}_{y_0}$

$\bar{d}_{y_0} = -P_0 \bar{X}_0 \bar{C}$ WHERE

$\bar{P}_0 = \left[I - \bar{X}_0 \bar{A}^T (\bar{A} \bar{X}_0^2 \bar{A}^T)^{-1} \bar{A} \bar{X}_0 \right]$

CONT'D

LEO COCO CONT'D:

$$\bar{C} = \begin{bmatrix} -20 \\ -10 \\ 0 \\ 0 \end{bmatrix} \quad \bar{A} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A^T IN SCILAB $\Rightarrow A'$ - SINGLE QUOTE

$\bar{X}^{-1} \Rightarrow \bar{X}$^-1 in scilab

$$\bar{P}_0 = \begin{bmatrix} 0.3157895 & -0.1053 & -0.2632 & -0.3684 \\ -0.1053 & 0.3684 & 0.4211 & -0.2105 \\ -0.2632 & 0.4211 & 0.5526 & -0.0263 \\ -0.3684 & -0.2105 & -0.0263 & 0.7632 \end{bmatrix}$$

$$\underline{\bar{d}_{y_0} = -\bar{P}_0 \bar{X}_0 \bar{C}_0}$$

$$\bar{d}_{y_0} = \begin{bmatrix} 4.2105 \\ 5.2632 \\ 3.1579 \\ -11.5789 \end{bmatrix} \quad \|\bar{d}_{y_0}\| = 13.7649 \quad \bar{y}_1 = \bar{y}_0 + \alpha \frac{\bar{d}_{y_0}}{\|\bar{d}_{y_0}\|} \Rightarrow \bar{y}_1 = \begin{bmatrix} 1.2906 \\ 1.3632 \\ 1.2179 \\ 0.2009 \end{bmatrix}$$

X_1

$\Rightarrow$ XFORM BACK : $\bar{y}_1 \rightarrow \bar{x}_1$

$$\bar{x}_1 = \bar{X}_0 \bar{y}_1 = \begin{bmatrix} 1.2906 \\ 2.7265 \\ 2.4359 \\ 0.4017 \end{bmatrix} \quad \bar{X}_1 = \begin{bmatrix} 1.2906 & & & \\ & 2.7265 & & 0 \\ & 0 & 2.4359 & \\ & & & 0.4017 \end{bmatrix}$$

$$\Rightarrow \bar{P}_1 = I - \bar{X}_1 A^T (\bar{A}\bar{X}_1^2 \bar{A}^T)^{-1} \bar{A}\bar{X}_1 \quad \bar{P}_1 = \begin{bmatrix} .1387 & -.1861 & -.2817 & -.0739 \\ -.1861 & .2703 & .4011 & -.0414 \\ -.2817 & .4011 & .5983 & -.0071 \\ -.0739 & -.0414 & -.0071 & .9927 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \bar{d}_{y_1} = -\bar{P}_1 \bar{X}_1 \bar{C} = \begin{bmatrix} -1.4932 \\ 2.5674 \\ 3.6648 \\ -3.0344 \end{bmatrix} \quad \|\bar{d}_{y_1}\| = 5.60888 \Rightarrow \bar{y}_2 = \bar{y}_1 + \alpha \frac{\bar{d}_{y_1}}{\|\bar{d}_{y_1}\|}$$

IS THIS A UNIT VECTOR OR IS IT $\bar{y}_1$?

X_2

$$\Rightarrow \bar{y}_2 = \begin{bmatrix} 0.7471123 \\ 1.4348491 \\ 1.6207111 \\ 0.4860235 \end{bmatrix}$$

X-FORM BACK TO X SPACE:

$$\bar{x}_2 = \bar{X}_1 \bar{y}_2 = \begin{bmatrix} 0.9642182 \\ 3.9120919 \\ 3.9478737 \\ 0.1952536 \end{bmatrix}$$

$$\bar{X}_2 = \begin{bmatrix} 0.9642182 & 0 & 0 & 0 \\ 0 & 3.9120919 & 0 & 0 \\ 0 & 0 & 3.9478737 & 0 \\ 0 & 0 & 0 & 0.1952536 \end{bmatrix} \quad \bar{P}_2 = I - \bar{X}_2 \bar{A}^T (\bar{A}\bar{X}_2^2 \bar{A}^T)^{-1} \bar{A}\bar{X}_2$$

CONTINUED $\Rightarrow$

$$\bar{P}_2 = \begin{bmatrix} 0.4001727 & -0.2947876 & -0.3900482 & -0.0340039 \\ -0.2947876 & 0.2192011 & 0.2892125 & -0.0246660 \\ -0.3900482 & 0.2892125 & 0.3818555 & -0.0161375 \\ -0.0340039 & -0.0246660 & -0.0161375 & -0.9979708 \end{bmatrix} \Rightarrow \bar{d}_{y_2} = -\bar{P}_2 \bar{X}_2 \bar{c}$$

$$\bar{d}_{g_2} = \begin{bmatrix} -3.7998574 \\ 2.8905566 \\ 3.7924249 \\ 1.6207021 \end{bmatrix} \qquad \|\bar{d}_{y_2}\| = 6.308993 \Rightarrow \bar{y}_3 = \bar{y}_0 + \alpha \frac{\bar{d}_{y_2}}{\|\bar{d}_{y_2}\|}$$

X_3

$$\bar{y}_3 = \begin{bmatrix} 0.4278224 \\ 1.4352563 \\ 1.5710584 \\ 0.7559568 \end{bmatrix} \qquad \bar{X}_3 = \bar{X}_2 \bar{y}_3 \Rightarrow \begin{bmatrix} 0.4125141 \\ 5.6148544 \\ 6.2023403 \\ 0.1476033 \end{bmatrix}$$

$$\bar{X}_3 = \begin{bmatrix} 0.4125141 & 0 & 0 & 0 \\ 0 & 5.6148544 & 0 & 0 \\ 0 & 0 & 6.2023403 & 0 \\ 0 & 0 & 0 & 0.1476033 \end{bmatrix} \Rightarrow \bar{P}_3 = I - \bar{X}_3 \bar{A}^T (\bar{A} \bar{X}_3^2 \bar{A}^T)^{-1} \bar{A} \bar{X}_3$$

$$P_3 = \begin{bmatrix} 0.8394893 & -0.1966451 & -0.2374443 & -0.0108201 \\ -0.1966451 & 0.0439692 & 0.0528832 & -0.0238736 \\ -0.2374443 & 0.0528832 & 0.0636664 & -0.0208927 \\ -0.0108201 & -0.0238736 & -0.0208927 & -0.9988752 \end{bmatrix} \Rightarrow \bar{d}_{y_3} = -\bar{P}_3 \bar{X}_3 \bar{c}$$

$$\bar{d}_{y_3} = \begin{bmatrix} -3.6697984 \\ 0.8464271 \\ 1.0103297 \\ -1.4297383 \end{bmatrix} \qquad \|\bar{d}_{y_3}\| = 4.153165 \Rightarrow \bar{y}_4 = \bar{y}_0 + \alpha \frac{\bar{d}_{y_3}}{\|d_{y_3}\|}$$

X_4

$$\bar{y}_4 = \begin{bmatrix} 0.1605659 \\ 1.1936128 \\ 1.231104 \\ 0.6729599 \end{bmatrix} \qquad \bar{X}_4 = \bar{X}_3 \bar{y}_4 \Rightarrow X_4 = \begin{bmatrix} 0.0662357 \\ 6.7019618 \\ 7.6357262 \\ 0.0993311 \end{bmatrix}$$

$$\bar{X}_4 = \begin{bmatrix} 0.0662357 & 0 & 0 & 0 \\ 0 & 6.7019618 & 0 & 0 \\ 0 & 0 & 7.6357262 & 0 \\ 0 & 0 & 0 & 0.0993311 \end{bmatrix} \qquad \bar{P}_4 = I - \bar{X}_4 \bar{A}^T (\bar{A} \bar{X}_4^2 \bar{A}^T)^{-1} \bar{A} \bar{X}_4$$

CONTINUED ⇒

$$P_4 = \begin{bmatrix} 0.9979221 & -0.0295743 & -0.0346141 & -0.0008886 \\ -0.0295743 & 0.0010960 & 0.0012186 & -0.0147891 \\ -0.0346141 & 0.0012186 & 0.0013698 & -0.0129729 \\ -0.000886 & -.0147891 & -0.0129729 & 0.9996120 \end{bmatrix} \Rightarrow \bar{d}_{y_4} = -\bar{P}_4 \bar{X}_4 \bar{c}$$

$$\bar{d}_{y_4} = \begin{bmatrix} -0.6600980 \\ 0.0342789 \\ 0.0358130 \\ -0.9923371 \end{bmatrix} \quad \|\bar{d}_{y_4}\| = 1.192862 \Rightarrow \bar{y}_5 = \bar{y}_0 + \alpha \frac{\bar{d}_{y_4}}{\|\bar{d}_{y_4}\|}$$

X_5

$$\bar{y}_5 = \begin{bmatrix} 0.4742953 \\ 1.0272999 \\ 1.0285216 \\ 0.2096988 \end{bmatrix} \quad X_5 = \bar{X}_4 \bar{y}_5 \Rightarrow X_5 = \begin{bmatrix} 0.0314153 \\ 6.8849245 \\ 7.8535094 \\ 0.0208296 \end{bmatrix}$$

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find $\bar{X}_6$

$$\bar{X}_5 = \begin{bmatrix} 0.0314153 & 0 & 0 & 0 \\ 0 & 6.8849245 & 0 & 0 \\ 0 & 0 & 7.8535094 & 0 \\ 0 & 0 & 0 & 0.0208296 \end{bmatrix} \quad \bar{P}_5 = \bar{I} - \bar{X}_5 \bar{A}^T (\bar{A} \bar{X}_5^2 A^T)^{-1} \bar{A} \bar{X}_5$$

$$\bar{P}_5 = \begin{bmatrix} 0.9995568 & -0.0136824 & -0.0159933 & -0.0000838 \\ & 0.0001964 & 0.0002269 & -0.0030242 \\ \text{SYMMETRIC} & & 0.0002629 & -0.0026509 \\ & & & 0.9999838 \end{bmatrix} \Rightarrow \bar{d}_{y_5} = -\bar{P}_5 \bar{X}_5 \bar{c}$$

$$\bar{d}_{y_5} = \begin{bmatrix} -0.3139963 \\ 0.0049283 \\ 0.0055765 \\ -0.2082663 \end{bmatrix} \Rightarrow \|\bar{d}_{y_5}\| = 0.376861 \Rightarrow \bar{y}_6 = \bar{y}_0 + \alpha \frac{\bar{d}_{y_5}}{\|\bar{d}_{y_5}\|}$$

X_6

$$\bar{y}_6 = \begin{bmatrix} 0.2084708 \\ 1.0124234 \\ 1.0140574 \\ 0.4749975 \end{bmatrix} \Rightarrow X_6 = \bar{X}_5 \bar{y}_6 = \begin{bmatrix} 0.0065492 \\ 6.9704585 \\ 7.9639095 \\ 0.0098940 \end{bmatrix}$$

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find X_7

$$\bar{X}_6 = \begin{bmatrix} 0.0065492 & 0 & 0 & 0 \\ 0 & 6.9704585 & 0 & 0 \\ 0 & 0 & 7.936095 & 0 \\ 0 & 0 & 0 & 0.0098940 \\ 0 & & & \end{bmatrix} \quad \bar{P}_6 = \bar{I} - \bar{X}_6 \bar{A}^T (\bar{A} \bar{X}_6^2 A^T)^{-1} \bar{A} X_6$$

$$\bar{P}_6 = \begin{bmatrix} 0.9999812 & -0.0028186 & -0.003309 & -0.0000081 \\ & 0.0000100 & 0.0000111 & -0.0014194 \\ & \text{symm.} & 0.0000125 & -0.0012467 \\ & & & 0.9999964 \end{bmatrix} = \bar{d}_{y6} = -\bar{P}_6 \bar{X}_6 \bar{c}$$

$$\bar{d}_{y6} = \begin{bmatrix} -0.0654900 \\ 0.0003250 \\ 0.0003395 \\ -0.0989391 \end{bmatrix} \Rightarrow \|\bar{d}_{y6}\| = 0.118651 \Rightarrow \bar{y}_7 = y_0 + \alpha \frac{\bar{d}_{y6}}{\|\bar{d}_{y6}\|}$$

$$\bar{y}_7 = \begin{bmatrix} 0.4756431 \\ 1.0026024 \\ 1.0027185 \\ 0.2078267 \end{bmatrix} \Rightarrow \bar{X}_7 = \bar{X}_6 \bar{y}_7 \qquad \bar{X}_7 = \begin{bmatrix} 0.0031151 \\ 6.9885986 \\ 7.9576692 \\ 0.0020562 \end{bmatrix}$$

4.210526		
5.263158	dyo=	13.76494
3.157895		
-11.5789		

-1.49311		
2.567457	dy1=	5.609037
3.664833		
-3.03465		

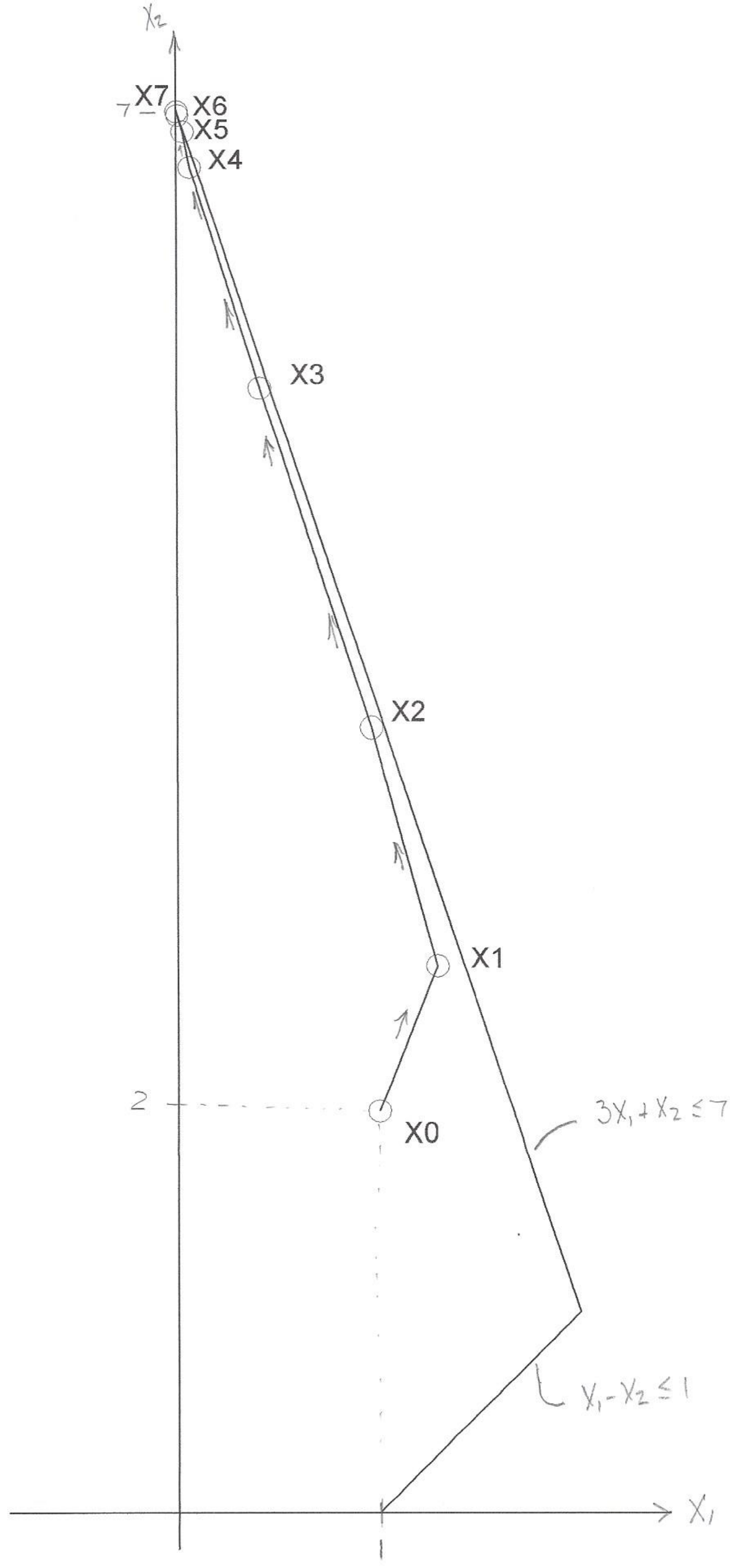
-3.79986		
2.890557	dy2=	6.308993
3.792425		
-1.6207		

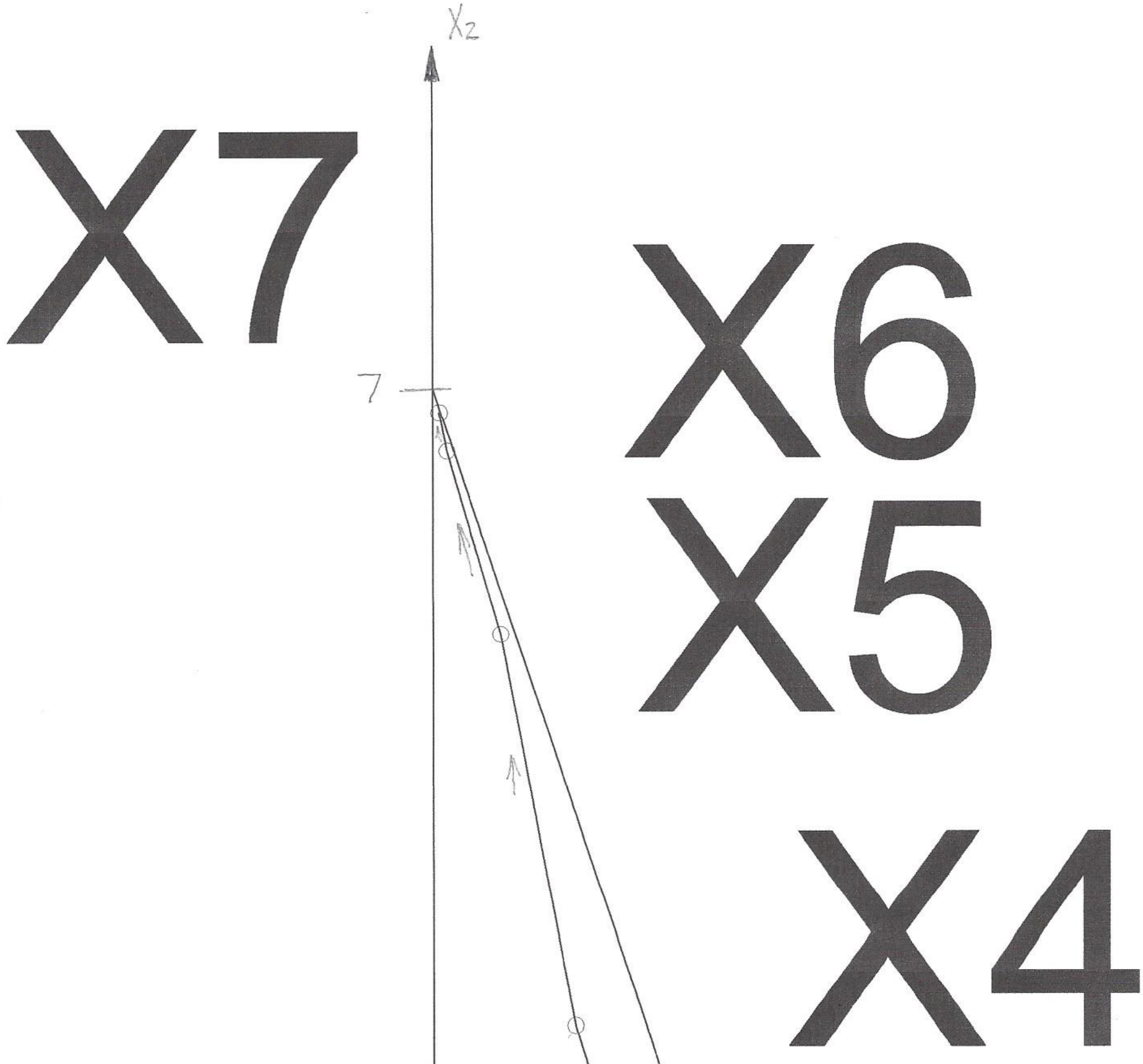
-3.6698		
0.846427	dy3=	4.153165
1.01033		
-1.42974		

-0.6601		
0.034279	dy4=	1.192862
0.035813		
-0.99234		

-0.314		
0.004928	dy5=	0.376861
0.005577		
-0.20827		

-0.06549		
0.000325	dy6=	0.118651
0.00034		
-0.09894		


X_2
7
X7
X6
X5
X4
X3
X2
X1
2
X0
$3X_1 + X_2 \le 7$
$X_1 - X_2 \le 1$
X_1



X_2
X7
X6
X5
X4
7
$3X_1+X_2 \le 7$

4.210526		1.290593	
5.263158	dyo= 13.76494	2.726483	x1= 3.897986
3.157895		2.43589	
-11.5789		0.401737	
-1.49311		0.964218	
2.567457	dy1= 5.609037	3.912092	x2= 5.64429
3.664833		3.947874	
-3.03465		0.195254	
-3.79986		0.412514	
2.890557	dy2= 6.308993	5.614854	x3= 8.377802
3.792425		6.20234	
-1.6207		0.147603	
-3.6698		0.066236	
0.846427	dy3= 4.153165	6.701962	x4= 10.16046
1.01033		7.635726	
-1.42974		0.099331	
-0.6601		0.031415	
0.034279	dy4= 1.192862	6.884925	x5= 10.4442
0.035813		7.853509	
-0.99234		0.02083	
-0.314		0.014943	
0.004928	dy5= 0.376861	6.902842	x6= 10.472
0.005577		7.874859	
-0.20827		0.004329	
-0.06549		0.003115	
0.000325	dy6= 0.118651	6.988599	x7= 10.5908
0.00034		7.957669	
-0.09894		0.002056	