

To Find $\bar{y}'$ From $\bar{y}^0$:

$$\bar{y}' = \bar{y}^0 + \alpha \frac{\bar{d}_y^0}{\|\bar{d}_y^0\|}$$

$$\bar{d}_y^0 = -\bar{P}_0 \bar{X}_0 \bar{c}$$

where $\bar{P}_k = [\bar{I} - \bar{X}_k \bar{A}^T (\bar{A} \bar{X}_k \bar{A}^T)^{-1} \bar{A} \bar{X}_k]$

$$\bar{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{X}_0 = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 13 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad \bar{c} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \bar{P}_0 = \begin{bmatrix} .3460 & .1278 & -.4578 & -.0197 \\ -.1278 & .9519 & .0894 & -.1464 \\ -.4578 & .0894 & .6795 & -.0138 \\ -.0197 & -.1464 & -.0138 & .0225 \end{bmatrix}$$

$$\bar{d}_y^0 = \begin{bmatrix} 6.6647 \\ 0.6516 \\ -7.3347 \\ -0.1003 \end{bmatrix}$$

To find $\bar{y}$ from $\bar{y} =$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{11.5}{11} = 1.045$$

$$\bar{y} = 1.045$$

$$P = [X^T A^{-1} (A^T X A)^{-1} A^T X - I] = Q$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = X \quad \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = I$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = Z \quad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = A$$

$$P = \begin{bmatrix} 1.000 & -0.577 & -0.577 & 0.577 \\ 0.577 & 1.000 & 0.577 & -0.577 \\ 0.577 & 0.577 & 1.000 & -0.577 \\ 0.577 & -0.577 & -0.577 & 1.000 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.577 & 0.577 & 0.577 & -0.577 \\ 0.577 & -0.577 & -0.577 & 1.000 \end{bmatrix}$$

$$\|\bar{d}_y^0\| = \sqrt{(6.6647)^2 + (.6516)^2 + (-9.3347)^2 + (-.1003)^2} = 11.48$$

letting $\alpha = .95$

$$\bar{y}' = \bar{y}^0 + \alpha \frac{\bar{d}_y^0}{\|\bar{d}_y^0\|}$$

$$\bar{y}' = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{.95}{11.48} \begin{bmatrix} 6.66 \\ .68 \\ -9.33 \\ -0.10 \end{bmatrix} = \begin{bmatrix} 1.55 \\ 1.056 \\ .226 \\ .99 \end{bmatrix}$$

TRANSFORM BACK

$$\bar{x}' = \bar{X}_0 \bar{y}' = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1.55 \\ 1.056 \\ .226 \\ .99 \end{bmatrix} = \begin{matrix} 15.5 \\ 2.10 \\ 1.58 \\ 12.87 \end{matrix}$$

$$\| \vec{b} \| = \sqrt{(2001)^2 + (-1003)^2 + (-8231)^2 + (-2010)^2} = 11418$$

$$\vec{b}' = \vec{b} / \| \vec{b} \|$$

$$\vec{b}' = \frac{1}{11418} \begin{bmatrix} 2001 \\ -1003 \\ -8231 \\ -2010 \end{bmatrix}$$

$$\begin{bmatrix} 22.1 \\ 220.1 \\ 20.0 \\ PF \end{bmatrix} = \begin{bmatrix} 22.2 \\ 82.0 \\ 28.3 \\ -0.10 \end{bmatrix} + \frac{27.0}{84.11} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \vec{b}'$$

TRANSFORM BACK

$$\begin{bmatrix} 22.1 \\ 220.1 \\ 20.0 \\ 22 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \vec{b}' = \vec{X}$$