Other aspects of the RCBD

- The RCBD utilizes an *additive model*
  - Can't explore interaction between treatments and blocks
- Treatments and/or blocks as random effects
  - Analysis is the same but interpretation is different
    - If blocks are random (e.g., selection of raw material batches) then assume treatment effect is the same throughout the *population* of blocks.
    - Any interaction between treatments and blocks are will not affect the test on treatment means (interaction will affect both treatment and error mean squares)

What about missing values?

- What happens if one of the measurements in your experiment is missing?
  - An error in measurement gives a result you know isn't right
  - Damage to a machine prevents you from completing a test
  - Etc.
- Inexact method – estimate the missing value and go on with the analysis
  - Reduce error degrees of freedom by 1 for each missing value
  - Danger – increase in "false" significance
- Exact method – general regression significance test
  - Test on *unbalanced* data (treatment and block not *orthogonal*)
  - Use Minitab or other computer application
    - in Minitab, use GLM model in ANOVA menu (2-Way ANOVA requires balanced designs)

Sample size

- Sample sizing in the RCBD refers to the number of blocks to run
- Can use Minitab sample size calculator with:
  - number of levels = treatment level
  - sample size = number of blocks
- Example 4.2, pg. 134
  - note the difference between the results using the OC curve approach and Minitab

The Latin square design

- Latin square designs are used to simultaneously control (or eliminate) two sources of nuisance variability
- Latin squares are not used as much as the RCBD in industrial experimentation
- A significant assumption is that the three factors (treatments, nuisance factors) do not interact
  - If this assumption is violated, the Latin square design will not produce valid results

The rocket propellant problem – A Latin square design

■ TABLE 4.8
Latin Square Design for the Rocket Propellant Problem

Batches of Raw Material	Operators 1	2	3	4	5
1	$A = 24$	$B = 20$	$C = 19$	$D = 24$	$E = 24$
2	$B = 17$	$C = 24$	$D = 30$	$E = 27$	$A = 36$
3	$C = 18$	$D = 38$	$E = 26$	$A = 27$	$B = 21$
4	$D = 26$	$E = 31$	$A = 26$	$B = 23$	$C = 22$
5	$E = 22$	$A = 30$	$B = 20$	$C = 29$	$D = 31$

- This is a *5x5 Latin square design*
- Latin letters (*A, B, C, D, E*) are the treatment levels
- The experiment is designed such that every treatment level is tested once at each combination of nuisance factors

Other Latin squares designs …

- 4 different fonts (treatments) tested for reading speed on different computer screens and in different ambient light levels (4 levels each)
- 3 different material types (treatments) tested for strength at different material lengths and measuring device (3 levels each)
- Note that once the design is complete, the order of the trials in the experiment is *randomized*

Statistical analysis of the Latin square design

- The statistical (effects) model is

$$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \begin{cases} i = 1, 2, \ldots, p \\ j = 1, 2, \ldots, p \\ k = 1, 2, \ldots, p \end{cases}$$

- The statistical analysis (ANOVA) is much like the analysis for the RCBD.
- See the ANOVA table, page 140
- Using Minitab (GLM) for the analysis …

■ **TABLE 4.9**

Analysis of Variance for the Latin Square Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}} = \frac{1}{p}\sum_{j=1}^{p} y_{.j.}^2 - \frac{y_{...}^2}{N}$	$p - 1$	$\frac{SS_{\text{Treatments}}}{p-1}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Rows	$SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i..}^2 - \frac{y_{...}^2}{N}$	$p - 1$	$\frac{SS_{\text{Rows}}}{p-1}$	
Columns	$SS_{\text{Columns}} = \frac{1}{p}\sum_{k=1}^{p} y_{..k}^2 - \frac{y_{...}^2}{N}$	$p - 1$	$\frac{SS_{\text{Columns}}}{p-1}$	
Error	SS_E (by subtraction)	$(p-2)(p-1)$	$\frac{SS_E}{(p-2)(p-1)}$	
Total	$SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$	$p^2 - 1$		

Checking for model adequacy

- The residuals in the Latin square are given by:

- As with any design, check the adequacy by plotting the appropriate residuals

Replication of Latin squares

- Small Latin squares provide a relatively small number of error degrees of freedom
  - makes seeing differences in treatment effects difficult
- Replication of Latin squares can be done in three ways:
  - Repeat each combination of row and column variable in each replication
  - Change 1 nuisance variable but keep the other the same for each replication
  - Use different levels of both nuisance variables
- Note, now the number of trials is $N = np^2$

Other designs

- Crossover designs
  - Repeated Latin squares in an experiment in which order (or time period) matters
  - See figure 4.7, pg. 145
- Graeco-Latin squares
  - Extension of the Latin squares design with 3 nuisance variables
    - Let each Greek letter indicate the 3rd nuisance factor level
    - Each combination of row variable, column variable,Greek letter, and Latin letter appears once and only once.
    - See example 4.4, pg. 147

Homework: Due Tuesday, 9/15

- 3.17
- 3.19
- 3.24

- 4.7
- 4.19
- 4.20
- 4.29