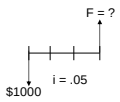
Single-Payment Factors (P/F, F/P)

Example: Invest $1000 for 3 years at 5% interest.



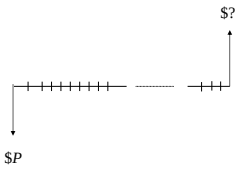
$F_1 = 1000 + (1000)(.05)$
$= 1000(1+.05)$

$F_2 = F_1 + F_1 i$
$= F_1 (1+i)$
$= 1000(1+.05)(1+.05)$
$= 1000(1+.05)^2$

$F_3 = 1000(1+.05)^3$

Single-Payment Factors (P/F, F/P)

Fundamental question: What is the future value, *F*, if a single present worth, *P*, is invested for *n* periods at an ROR of *i*% assuming compound interest?



General Solution:

Single-Payment Factors (P/F, F/P)

Fundamental questions:

1. What is the future value, *F*, if a single present worth, *P*, is invested for *n* periods at an ROR of *i*% assuming compound interest?

In general, $F = P(1+i)^n$

2. What is the present value, *P*, if a future value, *F*, is desired, assuming *P* is invested for *n* periods at *i*% compound interest?

$P = F/(1+i)^n$

Single-Payment Factors (P/F, F/P)

Standard Notation:

If wanting to know F given some P is invested for n periods at i% interest use –

$(F/P,i,n)$

Example: if i = 5% per month, n = 6 months, _____

If wanting to know P given some F if P is to be invested for n periods at i% interest use –

$(P/F,i,n)$

Example: if i = 7.5% per year, n = 4 years, _____

Single-Payment Factors (P/F, F/P)

- Standard Notation Equation: To find the value of F given some P is invested for n periods at i% interest use the equation –

$$F = P(F/P,i,n)$$

To find the value of P given some F if P is to be invested for n periods at i% interest use –

$$P = F(P/F,i,n)$$

The compound interest factor tables on pages 581-609 provide factors for various combinations of i and n.

Single-Payment Factors (P/F, F/P)

Example: If you were to invest $2000 today in a CD paying 8% per year, how much would the CD be worth at the end of year four?

F = $2000($F/P$,8%,4)

F = $2000(_____) from pg. 593

F = $2721

or,

$F = \$2000(1.08)^4$

F = $2000(1.3605)

F = $2721

Single-Payment Factors (P/F, F/P)

Example: How much would you need to invest today in a CD paying 5% if you needed $2000 four years from today?

P = $2000($P$/$F$,5%,4)

P = $2000(_____) from pg. 590

P = $1645.40

or,

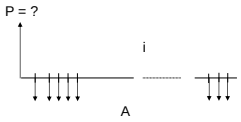
P = $2000/(1.05)^4

P = $2000/(1.2155)

P = $1645.40

Uniform Series Present Worth (P/A, A/P)

To answer the question: what is P given equal payments (installments) of value A are made for n periods at i% compounded interest?



Note: the first payment occurs at the end of period 1.

Examples?

- Reverse mortgages
- Present worth of your remaining car payments

Uniform Series Present Worth (P/A, A/P)

To answer the question: what is P given equal payments (installments) of value A are made for n periods at i% compounded interest?

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Standard Notation: (P/A,i,n)

Uniform Series Present Worth (P/A, A/P)

To answer the related question: what is *A* given *P* if equal installments of *A* are made for *n* periods at *i*% compounded interest?

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Standard Notation: (*A*/*P*,*i*,*n*)

Examples?

•Estimating your mortgage payment

Uniform Series Present Worth (P/A, A/P)

Example: What is your mortgage payment on a $90K loan if you are quoted 6.25% interest for a 30 year loan. (Remember to first convert to months.)

P = $90,000

i = _____

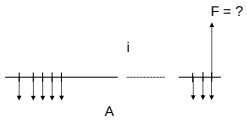
n = _____

A =

A = _____

Uniform Series Future Worth (*F/A, A/F*)

To answer the question: What is the future value at the end of year *n* if equal installments of $*A* are paid out beginning at the end of year 1 through the end of year *n* at *i*% compounded interest?



Uniform Series Future Worth (F/A, A/F)

Knowing:

$$P = F/(1+i)^n$$

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Then:

$$A = \frac{F}{(1+i)^n}\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

and,

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] \qquad F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Uniform Series Future Worth (F/A, A/F)

Example: If you invest in a college savings plan by making equal and consecutive payments of $2000 on your child's birthdays, starting with the first, how much will the account be worth when your child turns 18, assuming an interest rate of 6%?

A = \$2000, i = 6%, n = 18, find F.

F = 2000(F/A,6%,18)
F = \$2000(30.9057)
F = \$61,811.40

or,

Non-Uniform Cash Flows

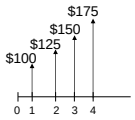
For example:

You and several classmates have developed a keychain note-taking device that you believe will be a huge hit with college students and decide to go into business producing and selling it.

1) Sales are expected to start small, then increase steadily for several years.
2) Cost to produce expected to be large in first year (due to learning curve, small lot sizes, etc.) then decrease rapidly over the next several years.

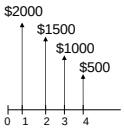
Arithmetic Gradient Factors (*P/G, A/G*)

Cash flows that increase or decrease by a constant amount are considered *arithmetic gradient* cash flows. The amount of increase (or decrease) is called the *gradient.*



G = $25

Base = $100

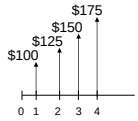


G = -$500

Base = $2000

Arithmetic Gradient Factors (*P/G, A/G*)

Equivalent cash flows:



=>

+

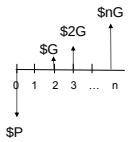
G = $25

Base = $100

Note: the gradient series by convention starts in year 2.

Arithmetic Gradient Factors (*P/G, A/G*)

To find P for a gradient cash flow that starts at the end of year 2 and end at year n:



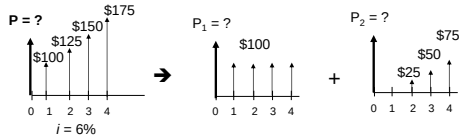
$$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

or $P = G(P/G,i,n)$

where $(P/G,i,n) = \frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$

Arithmetic Gradient Factors (*P/G, A/G*)

To find *P* for the arithmetic gradient cash flow:

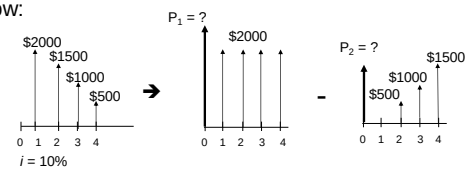


P_1 = _____ P_2 = _____

P = Base(P/A, i, n) + G(P/G, i, n) = _____

Arithmetic Gradient Factors (*P/G, A/G*)

To find *P* for the declining arithmetic gradient cash flow:

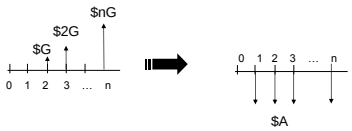


P_1 = _____ P_2 = _____

P = Base(P/A, i, n) - G(P/G, i, n) = _____

Arithmetic Gradient Factors (*P/G, A/G*)

To find the uniform annual series, *A,* for an arithmetic gradient cash flow *G*:



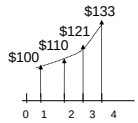
$A = G(P/G,i,n)\,(A/P,i,4)$

$= G(A/G,i,n)$

Where $(A/G,i,n) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$

Geometric Gradient Factors (P_g/A)

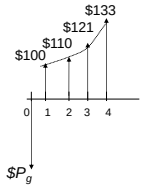
A Geometric gradient is when the periodic payment is increasing (decreasing) by a constant percentage:



A_1 = \$100, g = 0.1
A_2 = \$100(1+g)
A_3 = \$100(1+g)^2
A_n = \$100(1+g)$^{n-1}$

Geometric Gradient Factors (P_g/A)

To find the Present Worth, P_g, for a geometric gradient cash flow G:



$$P_g = A_1\left[\frac{1-\left(\frac{1+g}{1+i}\right)^n}{i-g}\right] \qquad g \neq i$$

$$P_g = A_1\left[\frac{n}{1+i}\right] \qquad g = i$$

Determining Unknown Interest Rate

To find an unknown interest rate from a single-payment cash flow or uniform-series cash flow, the following methods can be used:

1) Use of Engineering Econ Formulas
2) Use of factor tables
3) Spreadsheet (Excel)
   a) =IRR(first cell: last cell)
   b) =RATE(n,A,P,F)

Determining Unknown Interest Rate

Example: The list price for a vehicle is stated as $25,000. You are quoted a monthly payment of $658.25 per month for 4 years. What is the monthly interest rate? What interest rate would be quoted (yearly interest rate)?

Using factor table:

$25000 = $658.25(*P*/*A*,*i*,48) ➔ (*P*/*A*,*i*,48) = _____

i = _____ (HINT: start with table 1, pg. 581)

0r _____ annually

Determining Unknown Interest Rate

Example (cont'd)

Using formula:

$$\$25000 = \$658.25\left[\frac{(1+i)^{48}-1}{i(1+i)^{48}}\right]$$

$$37.9795 = \left[\frac{(1+i)^{48}-1}{i(1+i)^{48}}\right]$$

Use calculator solver or Excel trial and error method to find *i*.

Determining Unknown Number of Periods (n)

To find an unknown number of periods for a single-payment cash flow or uniform-series cash flow, the following methods can be used:

1)Use of Engineering Econ. Formulas.
2)Use of factor tables
3)Spreadsheet (Excel)
a) =NPER(*i*%,*A*,*P*,*F*)

Determining Unknown Number of Periods (n)

Example: Find the number of periods required such that an invest of $1000 at 5% has a future worth of $5000.

$P = F(P/F,5\%,n)$

$\$1000 = \$5000(P/F,5\%,n)$

$(P/F,5\%,n) =$ _____

$n =$ _____