Rate of Return Analysis

- Rate of Return (ROR) is the rate paid on the unpaid balance of borrowed money, or the rate earned on the unrecovered balance of an investment, so that the final payment or receipt brings the balance to exactly zero with considered interest.
- ROR is synonymous with: internal rate of return (IRR), return on investment (ROI), and profitability index (PI).

Rate of Return Analysis

To understand rate of return (ROR), consider the following: You have received a 5-year, 10% loan of $2000, with annual payments of A = $2000(A/P, 10%,5) = $527.59.

Year	Beginning Unrecovered Balance	Interest on Unrecovered Balance	Cash Flow	Recovered Amount	Ending Unrecovered Balance
0			($2,000)		($2,000.00)
1	($2,000.00)	$200	$527.59	$327.59	($1,672.41)
2	($1,672.41)	$167	$527.59	$360.35	($1,312.05)
3	($1,312.05)	$131	$527.59	$396.39	($915.66)
4	($915.66)	$92	$527.59	$436.03	($479.63)
5	($479.63)	$48	$527.59	$479.63	($0.00)

Calculating Rate of Return

- For an investment, the ROR can be determined by converting the series of cash flows into a Present Worth or Annual Worth equation.

- To develop the equation, set the PW (or AW) of all positive cash flows equal to the PW (or AW) of all negative cash flows.

$2000 = 527.59(P/A, *i*, 5)

Calculating Rate of Return

For example, what is the ROR of an investment of $1000 at time 0, that returns payments of $400 at the end of year one, and $800 at the end of year two.

$1000 = $400(P/F, i%,1) + $800(P/F,i%,2)

Using trial and error:

i%	(P/F,i%,1)	(P/F,i%,2)	$400(P/F,i%,1)+$800(P/F,i%,2)
8%	0.9259	0.8573	1056.2
9%	0.9174	0.8417	1040.32
10%	0.9091	0.8264	1024.76
11%	0.9009	0.8116	1009.64
12%	0.8929	0.7972	994.92

ROR = _____

Calculating Rate of Return

- In general, to find the ROR of an investment, set the PW of all disbursements (PW_D) equal to the PW of all receipts (PW_R) .

$$PW_D = PW_R$$

- Or, set the AW of all disbursements (AW_D) equal to the AW of all receipts (AW_R) .

$$AW_D = AW_R$$

Calculating Rate of Return

- The resulting ROR can be in the range of:
  - $-100\% < ROR < \infty$
  - An ROR of –100% means the entire investment is lost.

Accepting or rejecting a project.

If a project's ROR $\geq$ MARR, _____.

Your turn …

- Look at problem 7.5, pg. 264
  1. Draw the cash flow diagram
  2. Set up the equation (PW or AW)
  3. Determine the value of *i* that balances the equation.

Rate of Return

Cautions when performing an ROR analysis:

- Multiple ROR values
- Reinvestment at the ROR
- Computational Difficulty
- Special procedure for multiple alternatives

Because of these cautions, PW and AW analysis are often preferred to an ROR analysis. However, an ROR performed correctly leads to the same choice of projects as is found using PW and AW analysis.

Rate of Return

Multiple ROR values.

Consider the following cash flow:

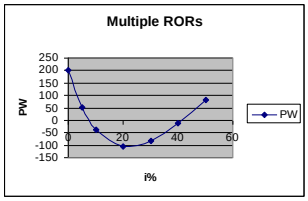
Year	0	1	2	3
Cash Flow	$2000	-$500	-$8100	$6800

PW = 2000 – 500(P/F,i%,1) – 8100(P/F,i%,2) + 6800(P/F,i%,3)

i%	0	5	10	20	30	40	50
PW	200	51.44	-39.55	-106.13	-82.01	-11.8	81.85

Rate of Return

i%	0	5	10	20	30	40	50
PW	200	51.44	-39.55	-106.13	-82.01	-11.83	81.85



PW equals 0 at i of approx. _____ and _____
Therefore ROR is either _____ or _____

Rate of Return

Multiple RORs:

In general, if there is more than one change in sign for cash flows, it is possible that there will be multiple ROR values in the –100% to infinity range.

To determine the correct ROR, use "common sense" or the method of the composite rate of return.

Bonds

Bonds are financial instruments of raising capital. In other words, in order to finance major projects, the government or corporations issue bonds in return for cash.

The borrower (corporation) promises to pay the face value of the bond upon maturity (V), and agrees to pay interest or dividends (I) at periodic times (c). Expected dividend payments are quarterly or semi-annually (c = 4 or 2). The dividend is determined using the stated bond coupon rate (b).

$$I = \frac{V * b}{c}$$

Bond Dividend

Example: Find the dividend paid on a $1000 US treasury bond stated as paying 5% quarterly.

V = $1000

b = 5%

c = 4

Therefore, I = _____

Rate of Return of a Bond

Suppose you are able to purchase a 10 year $1000 bond with a coupon rate of 8% with semi-annual interest payments. If you obtain the bond for $950, what is your nominal and effective annual interest rate.

I = _____ CFD:

n = _____

Using $PW_D = PW_A$, _____

PW	8%	6%	5%	4%
40	$392.73	$458.80	$498.49	$543.61
1000	$214.55	$311.80	$376.89	$456.39
Total	$607.27	$770.60	$875.38	$1,000.00

therefore i_{semi} = _____ r = _____

i_a = _____