Nominal -vs- Effective Interest Rates

Nominal interest rate, *r*, is an interest rate that does not include any consideration of compounding. This rate is often referred to as the Annual Percentage Rate (APR).

r = interest rate per period x number of periods

Effective interest rate is the actual rate that applies for a stated period of time. The *effective* interest rate is commonly expressed on an annual basis as the effective annual interest, i_a. This rate is often referred to as the Annual Percentage Yield (APY).

Nominal -vs- Effective Interest Rates

The following are nominal rate statements:

Nominal Rate (r) Time Period (t) Compounding Period (CP)

1) 12% interest per year, compounded monthly.

2) 12% interest per year, compounded quarterly.

3) 3% interest per quarter, compounded monthly.

What are the corresponding effective annual interest rates?

Nominal -vs- Effective Interest Rates

Corresponding effective annual interest rates:

Let *compounding frequency, m,* be the number of time the compounding occurs within the time period, *t*.

1) 12% interest per year, compounded monthly.

   $m = 12$

   Effective rate per *CP,* $i_{CP} = r/m = 1\%$ (per month).

   Effective annual rate, $i_e =$ _____

   In general, $i_e =$ _____

Nominal -vs- Effective Interest Rates

Corresponding effective annual interest rates:

2) 12% interest per year, compounded quarterly.
$m = 4$
Effective rate per *CP*, $i_{CP} = r/m =$ _____

Effective annual rate , $i_e =$ _____

3) 3% interest per quarter, compounded monthly.

$m =$ _____

Effective rate per *CP*, $i_{CP} =$ _____

Effective annual rate, $i_e =$ _____

Nominal -vs- Effective Interest Rates

Example: You are purchasing a new home and have been quoted a 15 year 6.25% APR loan. If you take out a $100,000 mortgage using the above rates, what is your monthly payment?

Compound period → _____

i_{CP} = _____

n = _____

A = _____

Determining *m*

- Given a stated APR and APY can you determine the compounding frequency?

  Example: A Certificate of Deposit has a stated APR of 8% with an Annual Yield of 8.3%. What is the compounding period?

Compound Period	**Effective Annual Interest**
1 day	_____
1 week	_____
1 month	_____
6 months	_____

Effective interest rates for any time period

- Let *PP* represent the payment period (period of time between cash flows.)

  And *m* is the number of compounding periods per payment period.

  - Effective $i = (1+r/m)^m - 1$
  - Where,

    r = nominal interest rate per payment period, *PP*.

    m = number of compounding periods per payment period.

Other Examples

- If cash flows are received on a semi-annual basis, what is the effective semi-annual interest rate under the following conditions:

a) 9% per year, compounded quarterly:

Effective i_{sa} = _____

b) 3% per quarter, compounded quarterly:

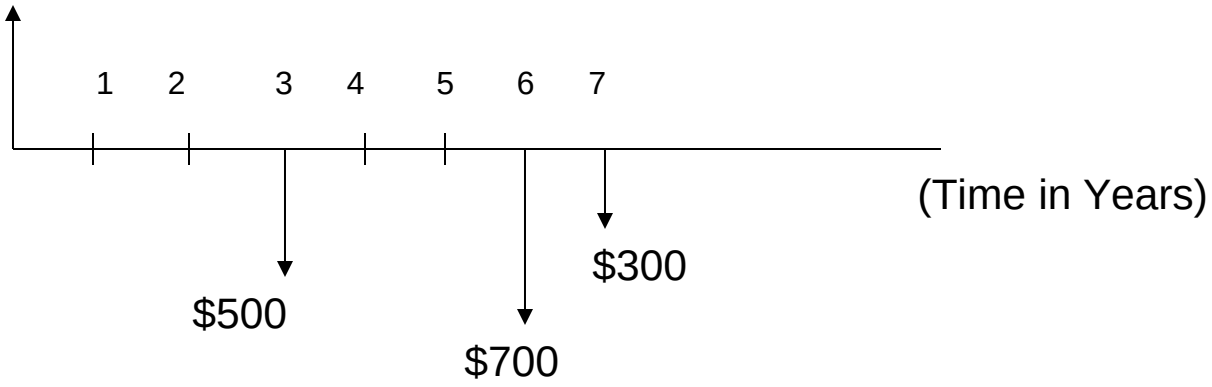
Effective i_{sa} = _____

c) 8.8% per year, compounded monthly.

Effective i_{sa} = _____

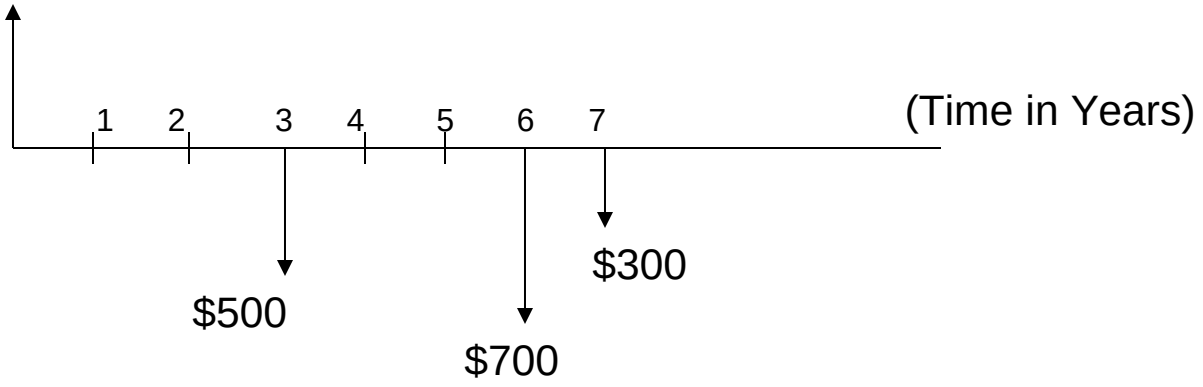
Equivalence Relations

- Example: Consider the following cash flow. Find the present worth if the cash flows earn a) 10% per year compounded quarterly, or b) 9% per year compounded monthly.



Equivalence Relations

Example: a) 10% per year compounded quarterly

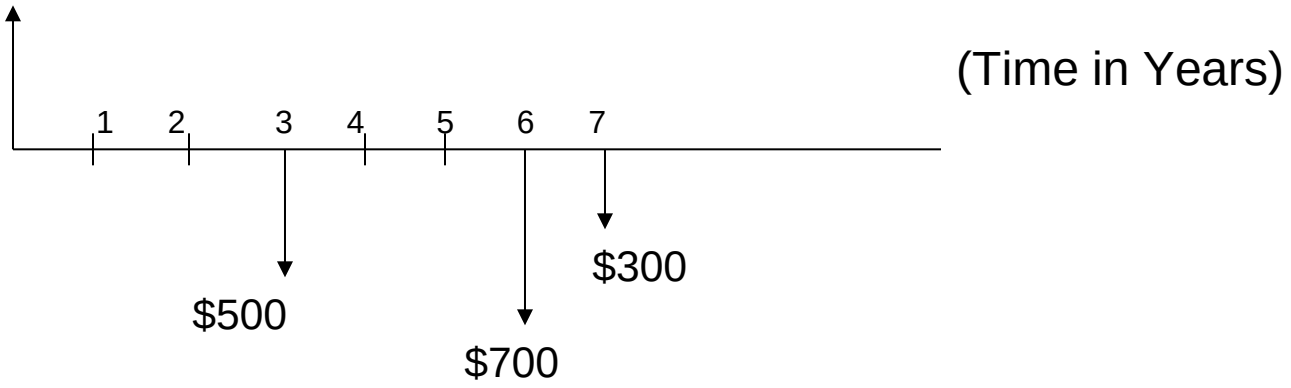


i_a = _____

P = _____

Equivalence Relations

Example: b) 9% per year compounded monthly.



i_a = _____

P = _____

Equivalence Relations (PP > CP)

Find *P* for the following in standard factor expressions :

Cash Flow	Interest Rate	Standard Notation
$500 semi–annually for 5 years	16% per year, compounded semi-annually	P =
$75 monthly for 3 years	24% per year, compound monthly	P =
$180 quarterly for 15 years	5% per quarter	P =
$25 per month increase for 4 years	1% per month	P =
$5000 per quarter for 6 years	1% per month	P =

Equivalence Relations (PP < CP)

- If payments occur more frequently than the compounding period, do these payments compound within the compounding period?

Answer: Depends

Equivalence Relations (PP < CP)

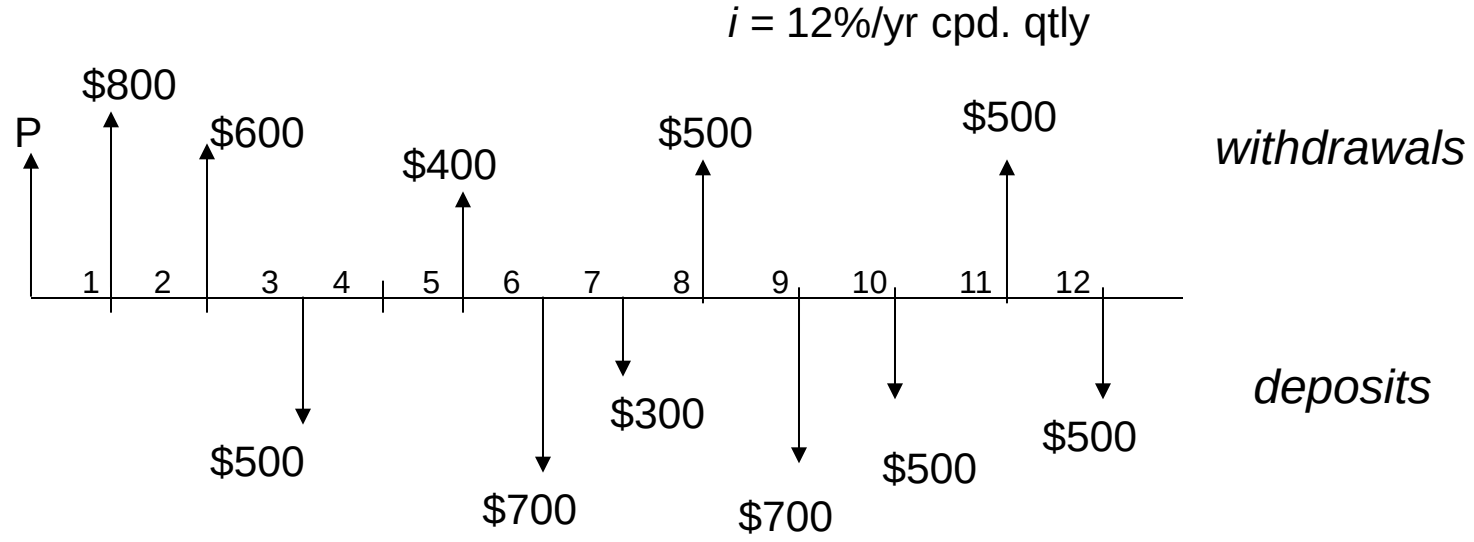
No compounding within compound period:

- All deposits are regarded as occurring at the end of the compounding period.
- All withdrawals are regarded as occurring at the beginning of the period.

Equivalence Relations (PP < CP)

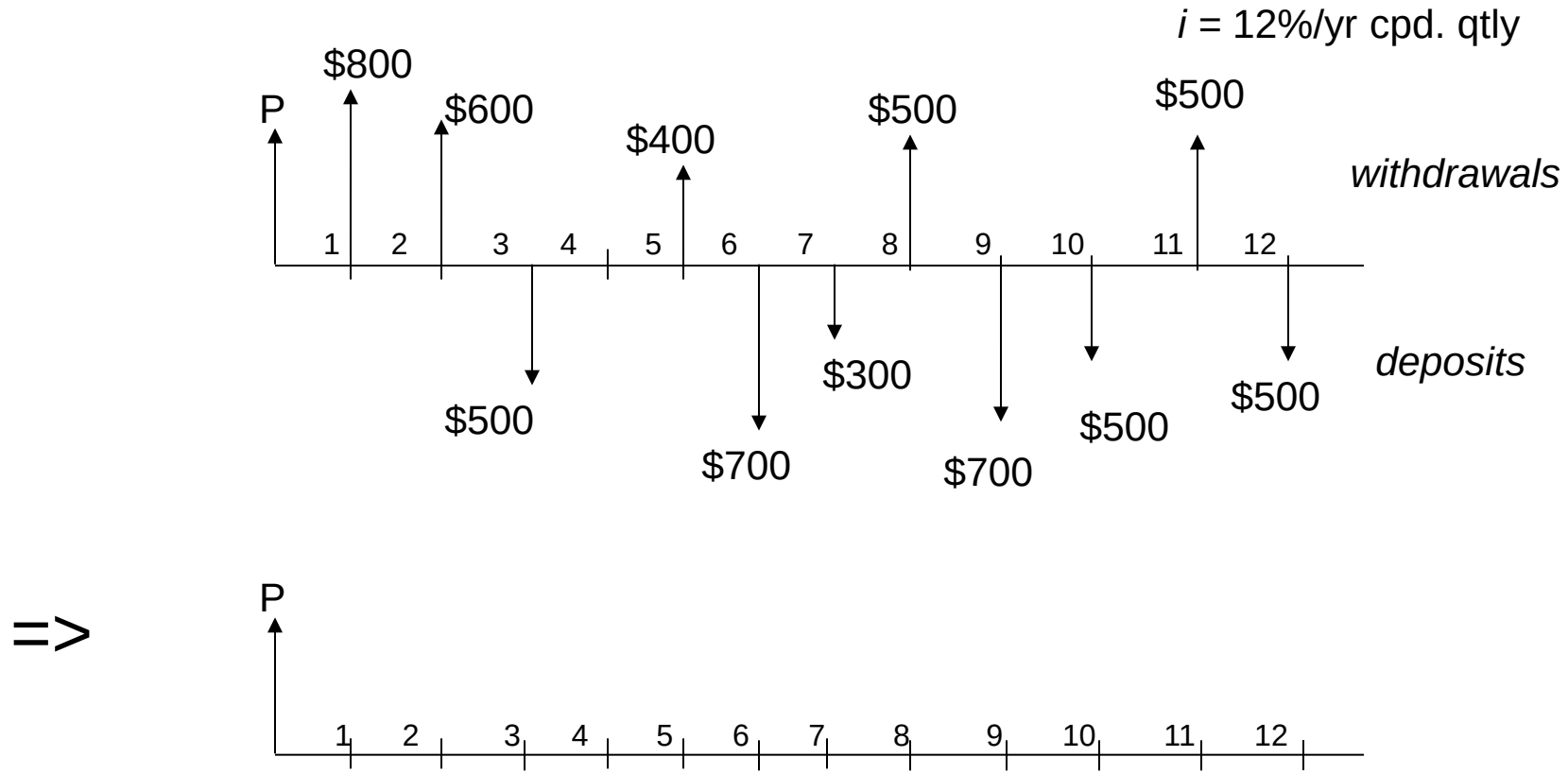
No compounding within compound period:

Example: A company has the following monthly cash flows. If the company expects an ROR of 12% per year, compounded quarterly, what is the present value of the cash flows?



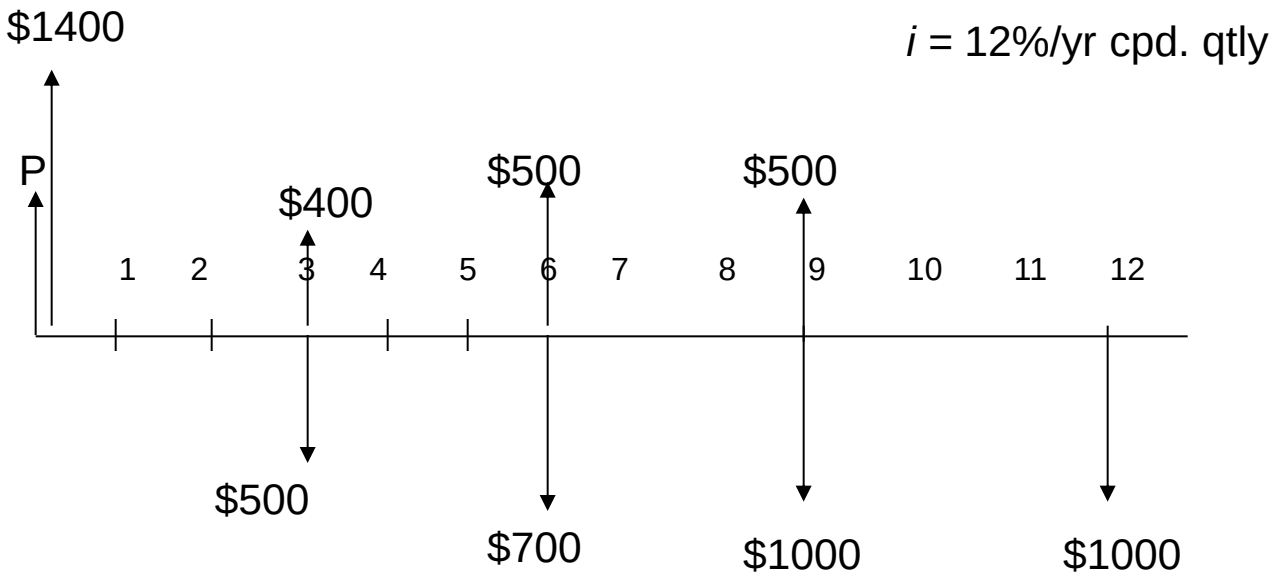
Equivalence Relations (PP < CP)

No compounding within compound period:



Equivalence Relations (PP < CP)

No compounding within compound period:



P = _____

(Hint: we can now look at quarters …)

Equivalence Relations (PP < CP)

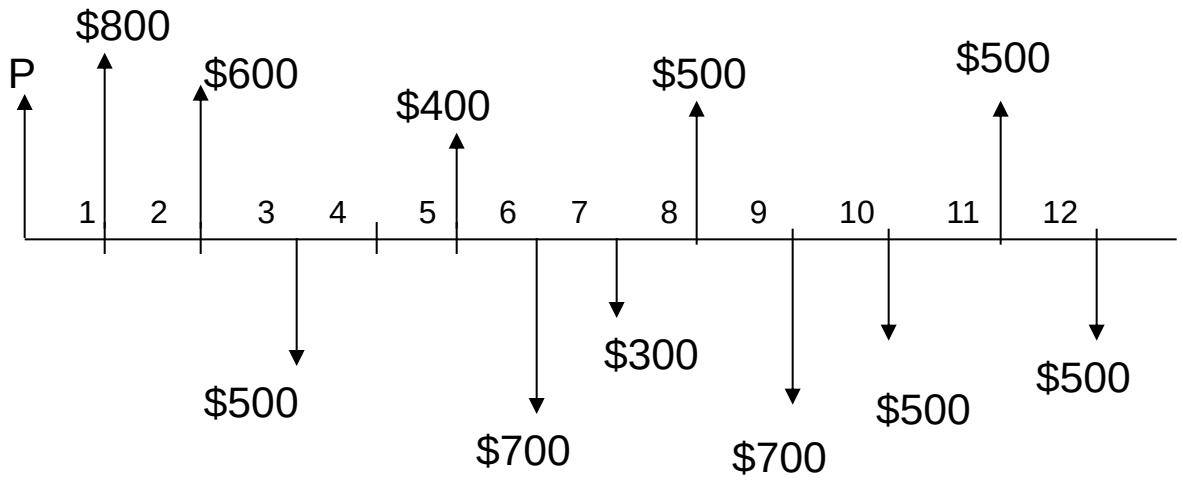
With interperiod compounding:

- If interest is compounded within the period, treat interest on cash flows the same as the treatment of nominal interest rates.

Equivalence Relations (PP < CP)

With interperiod compounding:

Example: A company has the following monthly cash flows. If the company expects an ROR of 12% per year, compounded quarterly, what is the present value of the cash flows?

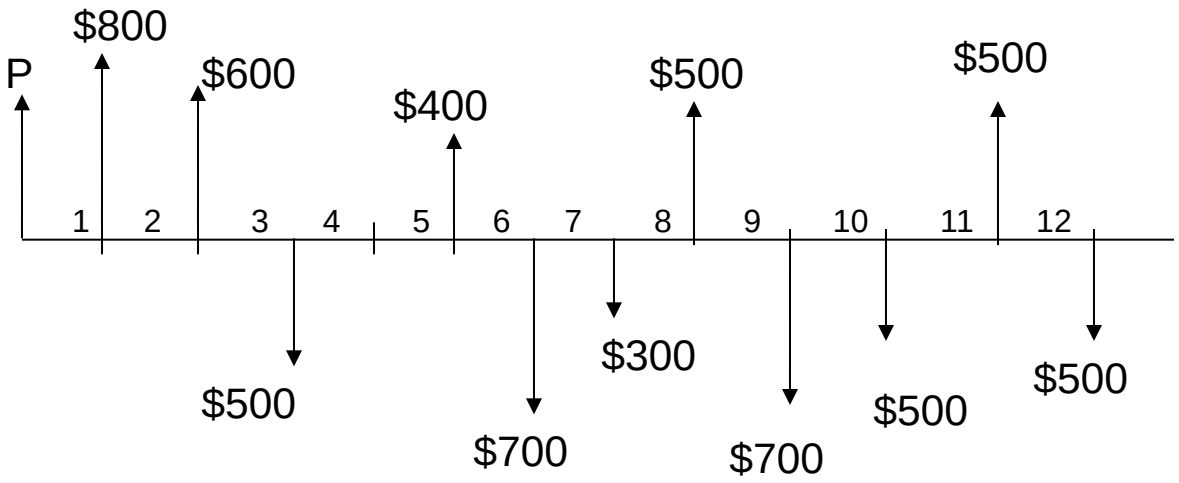


Equivalence Relations (PP < CP)

With interperiod compounding:

For our example: Interest is compounded monthly at the rate of 1%.

P = $800(P/F,1%,1) + $600(P/F,1%,2) - $500(P/F,1%,3) + etc….



Continuous compounding

- Recall,

  Effective $i = (1+r/m)^m - 1$

  Where m = number of compounding periods per payment period.

- As m approaches infinity,

  – $i = e^r - 1$

- Example: A 15% APR compounded continuously is effectively:

  $i =$ _____