Composite Reinforcement of Cylindrical Pressure Vessels

Cylindrical Pressure Vessels

Cylindrical pressure vessels are in widespread use for a variety of applications

- SCBA and SCUBA tanks
- Propane tanks
- Compressed Natural Gas (CNG) and hydrogen for Alternative Fuel Vehicles
- Medical oxygen tanks
- Laboratory gas tanks

Depending on the application, primary design considerations include:

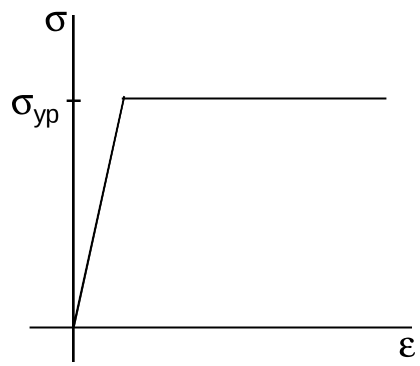
- Weight
- Cost
- Pressure capacity
- Storage capacity
- Safety and durability

Design and Analysis Considerations

First, consider a metallic, thin-walled cylindrical vessel

For preliminary design/analysis, and for today's discussion, we will restrict ourselves to the following conditions and assumptions:

- Vessels are thin-walled ($t < R/10$)
  - Stresses are uniform through the wall thickness (membrane loading)
  - Stress normal to the wall thickness is much less than membrane stresses
- Material (typically steel or aluminum) is elastic-perfectly plastic
  - von Mises yield criterion applies
- We will consider the cylinder portion only
  - End closures (domes) are beyond today's scope



Note that the vessel is axisymmetric about cylinder axis

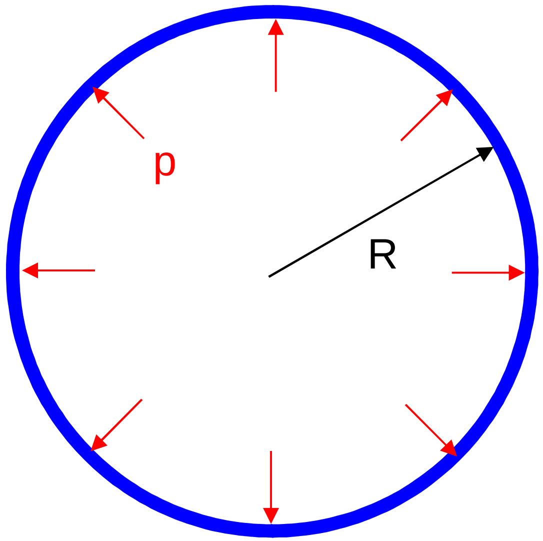
Applied pressure loading is also axisymmetric

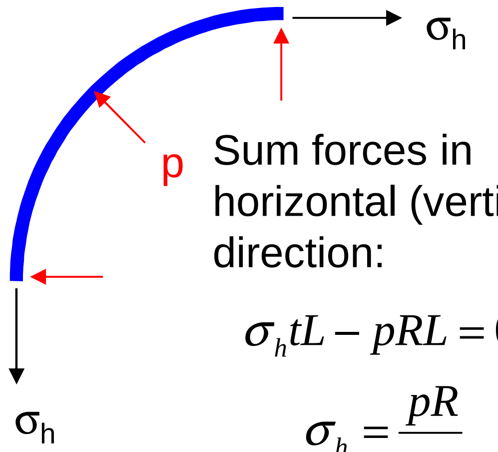
Equibrium in Hoop and Axial Directions

Notation

- p, internal pressure
- R, radius
- t, wall thickness
- σ, stress
- h, hoop direction
- a, axial direction

Consider a slice of length L:

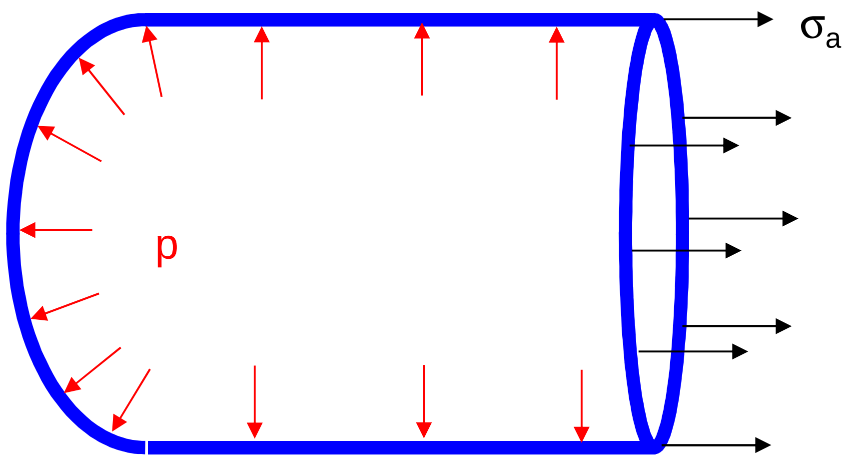




Sum forces in horizontal (vertical) direction:

$$\sigma_h tL - pRL = 0$$

$$\sigma_h = \frac{pR}{t}$$



Sum forces in horizontal direction:

$$\sigma_a (2\pi R)t - p(\pi R^2) = 0$$

$$\sigma_a = \frac{pR}{2t}$$

Summary of Stresses

Biaxial state of stress

- Normal stress in the hoop direction

$$\sigma_h = \frac{pR}{t}$$

- Normal stress in the axial direction

$$\sigma_a = \frac{pR}{2t}$$

- No shear stress

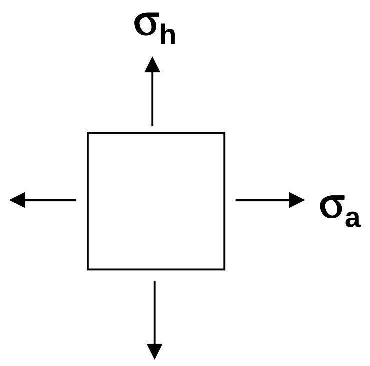
$$\tau_{ah} = 0$$

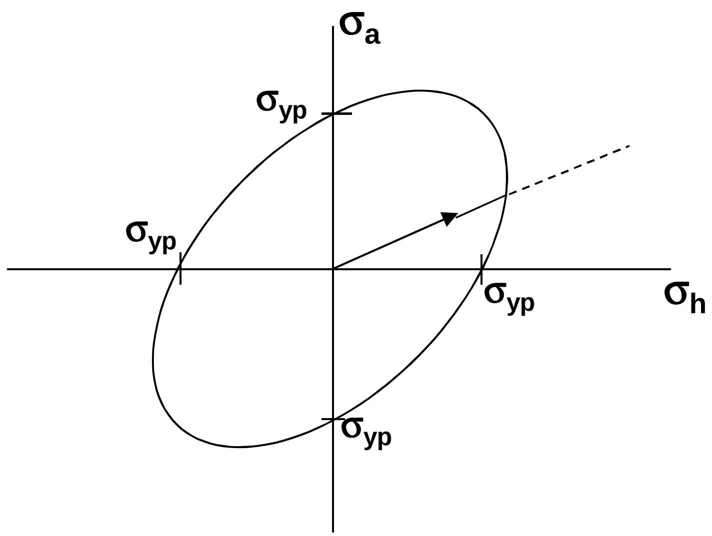
- Therefore, σ_h and σ_a are principal stresses

- von Mises yield criterion in two dimensions:

$$\sigma_{yp}^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$

$$\sigma_{yp}^2 = \sigma_h^2 - \sigma_h\sigma_a + \sigma_a^2$$





- Failure occurs when load line reaches the von Mises ellipse

Design Equation

Failure criterion:

$$\sigma_{yp}^2 = \sigma_h^2 - \sigma_h \sigma_a + \sigma_a^2$$

Substitute for the hoop and axial stresses, set *p* = *p_f* , and simplify:

$$\sigma_{yp}^2 = \left(\frac{p_f R}{t}\right)^2 - \left(\frac{p_f R}{t}\right)\left(\frac{p_f R}{2t}\right) + \left(\frac{p_f R}{2t}\right)^2 = \left(\frac{p_f R}{t}\right)^2 \left(1 - \frac{1}{2} + \frac{1}{4}\right)$$

$$\sigma_{yp} = \frac{\sqrt{3}}{2} \frac{p_f R}{t}$$

Example

Given:

- Tank Dimensions
  - Diameter = 12 in.
  - Length of cylinder section = 3 ft. = 36 in.
- Load
  - Service pressure = 3600 psi
  - Factor of safety against burst = 2.25
- Material is 4130 steel
  - E = 30 x 10^6 psi
  - Poisson's ratio, ν = 0.25
  - Yield stress, σ_{yp} = 112,000 psi
  - Weight density, ρ = 0.283 lb/in^3

Determine the required wall thickness

Pressure at failure:

- **p_f = (3600)2.25 = 8100 psi**

$$\sigma_{yp} = \frac{\sqrt{3}}{2}\frac{p_f R}{t}$$

$$\Rightarrow t = \frac{\sqrt{3}}{2}\frac{p_f R}{\sigma_{yp}} = \frac{\sqrt{3}}{2}\frac{(8100)(6)}{112{,}000}$$

$$t = 0.376\ in.$$ **(< R/10)**

Weight of tank (cylinder section):

$$W = \rho V = \rho(2\pi R)tL$$

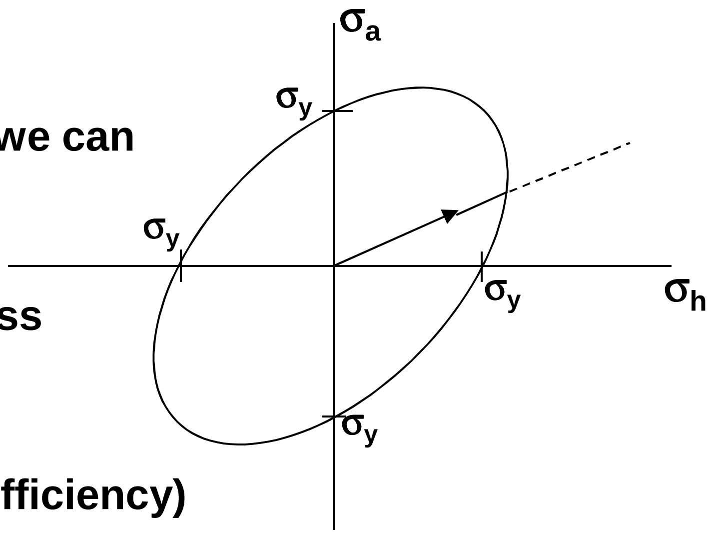
$$W = 0.283(2\pi 6)(0.376)(36)$$

$$W = 144\ lb.$$

Some Observations

$$\sigma_h = \frac{pR}{t} \qquad \sigma_a = \frac{pR}{2t}$$

The hoop stress is twice as large as the axial stress

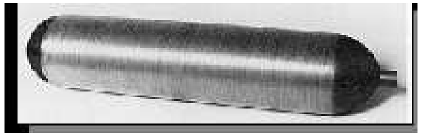


If we apply reinforcement in the hoop direction, maybe we can

- **Reduce the tank load in the hoop direction**
- **Make hoop stress more nearly equal to the axial stress**
- **Enable reduction in tank wall thickness**
- **Reduce the weight of the tank (improved structural efficiency)**

Wrap the cylinder section with continuous fiber reinforced composite material

- **Common materials are glass or carbon fibers in an epoxy matrix**
- **With all reinforcing fibers in the hoop direction, we will assume that the reinforcement carries load in the hoop direction only**
- **Assume that the composite is linearly elastic to failure**

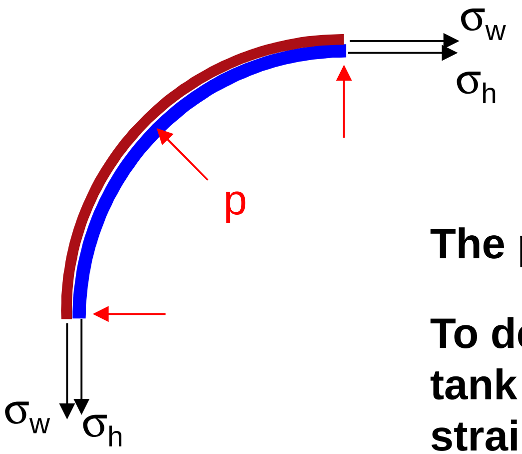


Equilibrium Considerations

Notation: subscript w denotes the hoop-wrap reinforcement

Equilibrium in the axial direction is unchanged $\sigma_a = \frac{pR}{2t}$

Equilibrium in the hoop direction:



$$(t\sigma_h + t_w\sigma_w)L - pRL = 0$$

$$\Rightarrow t\sigma_h + t_w\sigma_w = pR$$

The problem is now statically indeterminate

To determine how the hoop load is divided between the tank and the wrap, we need an additional equation: hoop strain in tank and wrap must be equal

As long as the tank has not yielded, Hooke's law applies:

$$\varepsilon_h = \frac{1}{E}(\sigma_h - \upsilon\sigma_a) \qquad \varepsilon_w = \frac{1}{E_w}\sigma_w$$

$$\Rightarrow \frac{1}{E}(\sigma_h - \nu\sigma_a) = \frac{1}{E_w}\sigma_w$$

Solution to the Elastic Equations

$$\sigma_a = \frac{pR}{2t}$$

$$t\sigma_h + t_w\sigma_w = pR$$

$$\Downarrow$$

$$\sigma_w = \frac{pR - t\sigma_h}{t_w}$$

$$\frac{1}{E}(\sigma_h - \nu\sigma_a) = \frac{1}{E_w}\sigma_w$$

$$\Downarrow$$

$$\Rightarrow \frac{1}{E}\left(\sigma_h - \nu\frac{pR}{2t}\right) = \frac{1}{E_w}\frac{pR - t\sigma_h}{t_w}$$

$$\sigma_a = \frac{pR}{2t}$$

(**equilibrium**)

$$\sigma_w = \frac{pR - t\sigma_h}{t_w}$$

(**equilibrium**)

and

$$\sigma_h = \frac{Et + \frac{\nu}{2}E_w t_w}{Et + E_w t_w}\frac{pR}{t}$$

(**Hooke's Law**)

These equations are valid as long as the wrap has not failed:

$$\sigma_w < \sigma_{fw}$$

and the tank has not yielded:

$$\sigma_{eq} = \sqrt{\sigma_h^2 - \sigma_h\sigma_a + \sigma_a^2} < \sigma_{yp}$$

Following Tank Yield

Once the tank has yielded (assuming the wrap is still intact):

$$\sigma_a = \frac{pR}{2t}$$

still, from axial equilibrium

But now the tank hoop stress is obtained from the yield criterion

$$\sigma_{yp}^{\ 2} = \sigma_h^2 - \sigma_h \sigma_a + \sigma_a^2 \Rightarrow \sigma_h^2 - \sigma_h \sigma_a + \sigma_a^2 - \sigma_{yp}^{\ 2} = 0$$

Solve for the hoop stress using the quadratic formula:

$$\sigma_h = \frac{\sigma_a + \sqrt{\sigma_a^2 - 4\left(\sigma_a^2 - \sigma_{yp}^2\right)}}{2} \Rightarrow \sigma_h = \frac{1}{2}\left[\frac{pR}{2t} + \sqrt{4\sigma_{yp}^2 - 3\left(\frac{pR}{2t}\right)^2}\,\right]$$

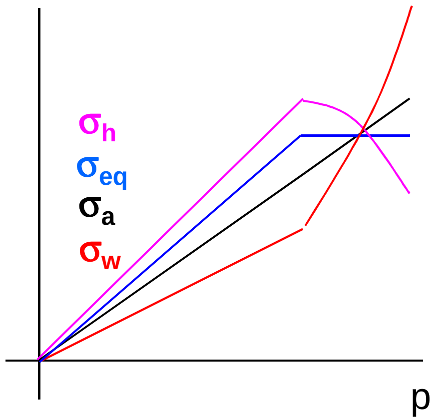
And get the stress in the wrap from equilibrium in the hoop direction:

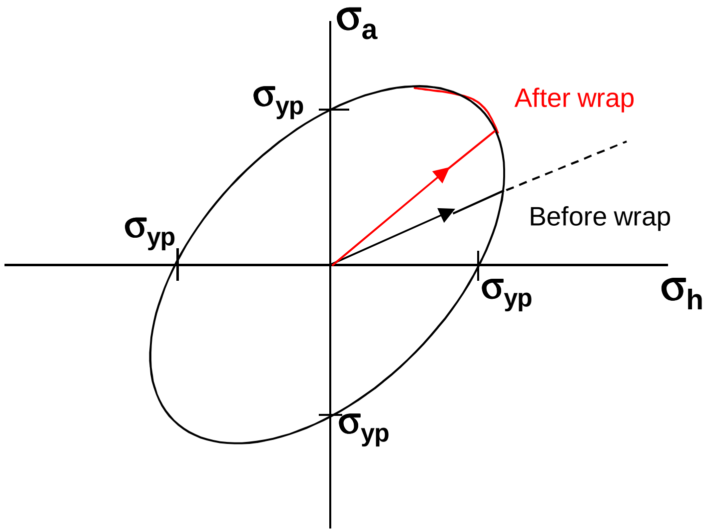
$$\sigma_w = \frac{pR - t\sigma_h}{t_w}$$

Note from the equation for σ_h, the value inside the square root must be ≥ 0:

$$4\sigma_{yp}^2 - 3\left(\frac{pR}{2t}\right)^2 \geq 0 \Rightarrow t \geq \frac{\sqrt{3}}{4}\frac{pR}{\sigma_{yp}}$$

What's it All Mean?





As the pressure increases up to tank yield, hoop stress and axial stress in the tank, as well as the stress in the wrap, increase linearly.

After the tank yields:

- Axial stress in the tank continues to increase linearly (axial equilibrium)
- Hoop stress decreases, keeping σ_h and σ_a on the von Mises ellipse (yield criterion)
- Stress in the wrap must increase at a faster rate (hoop equilibrium)

Two possible failure modes:

- Failure of the wrap (preferred)
- Failure of the tank

Example

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- Material is 4130 steel
  - E = 30 x 10^6 psi
  - Poisson's ration, ν = 0.25
  - Yield stress, σ_y = 112,000 psi
  - Weight density, ρ = 0.283 lb/in^3
- Hoop wrap with T300 carbon fiber/epoxy
  - E_w = 22 x 10^6 psi
  - Failure stress σ_{fw} = 270,000 psi
  - Weight density, ρ_ω = 0.056 lb/in^3

Determine thickness of tank and wrap

Minimum tank thickness

$$t \geq \frac{\sqrt{3}}{4}\frac{p_f R}{\sigma_{yp}} = \frac{\sqrt{3}}{4}\frac{(8100)(6)}{112{,}000} = 0.188\ in.$$

Set t = 0.200 in.

At burst pressure,

$$\sigma_a = \frac{p_f R}{2t} = \frac{(8100)(6)}{2(0.200)} = 121{,}500\ psi$$

Tank has yielded; use post-yield equations for σ_h and σ_a at burst pressure:

$$\sigma_h = \frac{1}{2}\left[\frac{p_f R}{2t} + \sqrt{4\sigma_y^2 - 3\left(\frac{p_f R}{2t}\right)^2}\right] = 99{,}121\ psi$$

$$\sigma_{fw} = \frac{p_f R - t\sigma_h}{t_w} \Rightarrow t_w = \frac{p_f R - t\sigma_h}{\sigma_{fw}} = 0.107\ in.$$

Weight of tank (cylinder section):

$$W = 2\pi RL(\rho t + \rho_w t_w)$$

$$W = (2\pi 6)(36)((0.283)(0.200) + (0.056)(0.107))$$

$$W = 85\ lb.$$

Closing Comments

Governing equations are readily entered into a spreadsheet

- Examine the effects of different material selections, thicknesses, tank geometries
- For a given tank radius, material selection, and pressure requirement find the thicknesses that give optimum design (minimum weight)

Minimum weight design is achieved by enforcing the condition that both tank and wrap fail simultaneously at the required burst pressure (82 lb. in previous example)

Additional considerations

- Compare pressure at which tank yields to service pressure (5618 psi vs. 3600 psi in previous example)
  - Autofrettage (intentionally pressurize beyond the elastic limit) to increase the elastic range and improve fatigue strength
- Thermal effects: tank and wrap have different coefficients of thermal expansion
  - Processing-induced residual stresses due to elevated temperature composite cure
  - Operation at elevated and reduced pressures

The next steps to reduce weight :

- Fully-wrapped with load-sharing metallic liner
- Fully-wrapped with plastic liner