Fundamental Equations of Dynamics

KINEMATICS

Particle Rectilinear Motion

General case	Constant $a = a_c$
$a = \frac{dv}{dt}$	$v = v_O + a_c t$
$v = \frac{ds}{dt}$	$s = s_O + v_O t + \frac{1}{2} a_c t^2$
$a = v\frac{dv}{ds}$	$v^2 = v_O^2 + 2a_c\left(s - s_O\right)$

Particle Curvilinear Motion

Rectangular	Tangential and normal
$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$	$\bar{v} = \dot{s}\hat{u}_t = v\hat{u}_t$
$\bar{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$	$\bar{a} = \dot{v}\hat{u}_t + \frac{v^2}{\rho}\hat{u}_n$
$\bar{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$	$\rho = \left\lvert \frac{\left[1+\left(dy/dx\right)^2\right]^{\frac{3}{2}}}{d^2y/dx^2} \right\rvert$

Radial and transverse

$\bar{r} = r\hat{u}_r$

$\bar{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$

$\bar{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta$

Relative Motion

$\bar{v}_B = \bar{v}_A + \bar{v}_{B/A}$

$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

General case	Constant $\alpha = \alpha_c$
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_O + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_O + \omega_O t + \frac{1}{2}\alpha_c t^2$
$a = v\frac{dv}{ds}$	$\omega^2 = \omega_o^2 + 2\alpha_c\left(\theta - \theta_o\right)$

For Point P

$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$

General Plane Motion - Translating Axes

$\bar{v}_B = \bar{v}_A + \bar{\omega} \times \bar{r}_{B/A}$

$\bar{a}_B = \bar{a}_A + \bar{\alpha} \times \bar{r}_{B/A} - \omega^2 \bar{r}_{B/A}$

KINETICS

Mass Moment of Inertia $I = \int r^2 dm$

Parallel Axis Thm.: $I = I_G + md^2$

Radius of Gyration: $k = \sqrt{\frac{I}{m}}$

Equations of Motion

Particle

$\bar{F} = m\bar{a}$

Rigid Body (Plane Motion)

$\bar{F} = m\bar{a}_G$ **G: center of mass;**

and $\bar{M}_G = I_G\bar{\alpha}$

or $\bar{M}_O = I_O\bar{\alpha}$ **(pure rotation about 0)**

or $\bar{M}_P = I_G\bar{\alpha} + \bar{r}_{G/P} \times m\bar{a}_G$

Principle of Work and Energy

$U_{1-2} = T_2 - T_1$

Kinetic Energy

Particle $T = \frac{1}{2}mv^2$

Rigid Body $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$, **or**

$T = \frac{1}{2}I_O\omega^2$ **(pure rotation about 0)**

Work

Definition $U = \int \bar{F} \cdot d\bar{r} = \int \bar{F} \cdot \bar{v}dt$

Gravity $U_g = -mg\left(y_2 - y_1\right)$

Spring $U_S = -\frac{1}{2}k\left(s_2^2 - s_1^2\right)$

Couple Moment $U_M = M\Delta\theta$

Conservation of Energy $T_1 + V_1 = T_2 + V_2$

Potential Energy

$V_g = mgy \qquad V_S = \frac{1}{2}ks^2$

Principle of Impulse and Momentum

Particle

$\int_{t_1}^{t_2} \bar{F}dt = m\bar{v}_2 - m\bar{v}_1$

Rigid Body

$\int_{t_1}^{t_2} \bar{F}dt = m\bar{v}_{G2} - m\bar{v}_{G1}$

Angular

$\int_{t_1}^{t_2} \bar{M}_G dt = I_G\bar{\omega}_2 - I_G\bar{\omega}_1$

or $\int_{t_1}^{t_2} \bar{M}_O dt = I_O\bar{\omega}_2 - I_G\bar{\omega}_1$ **(pure rotation about 0)**

Coefficient of Restitution

$e = -\frac{\left(v_{B2} - v_{A2}\right)}{\left(v_{B1} - v_{A1}\right)}$