Sixth Edition

CHAPTER

4

MECHANICS OF MATERIALS

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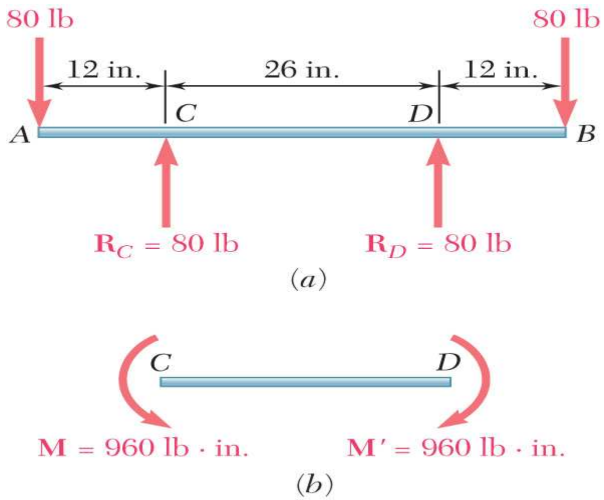
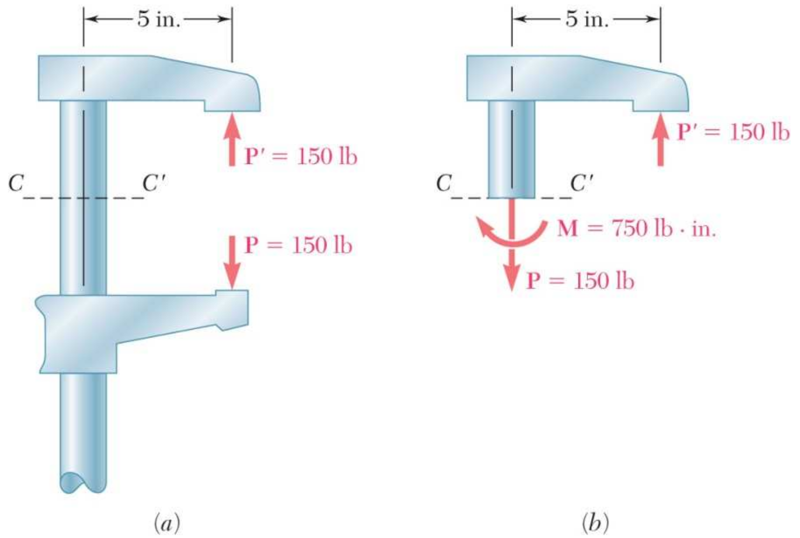
Pure Bending

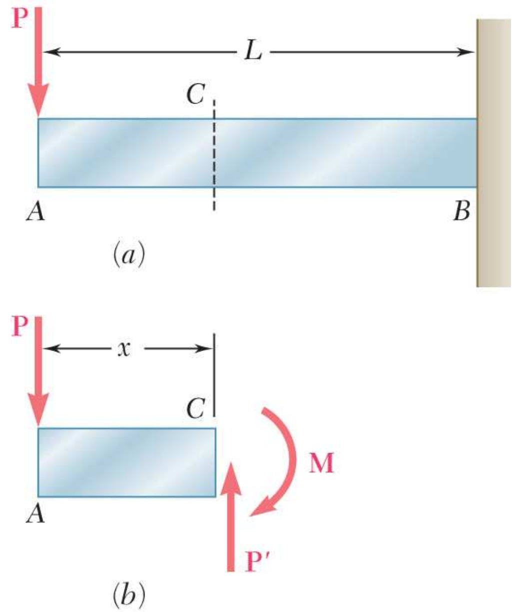
Pure Bending

Pure Bending:
Prismatic members subjected to equal and opposite couples acting in the same longitudinal plane



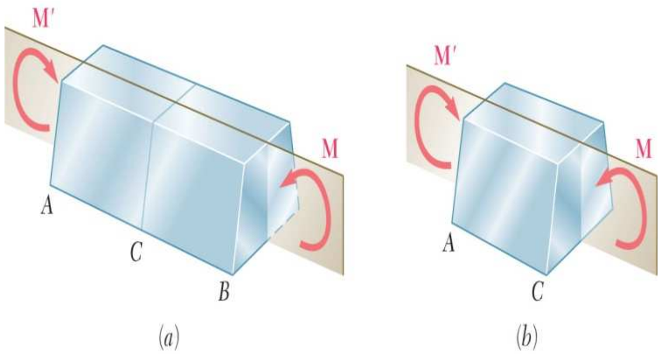
Other Loading Types



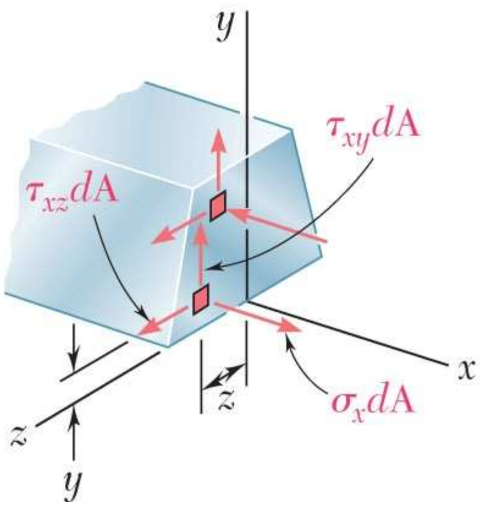


- *Eccentric Loading*: Axial loading which does not pass through section centroid produces internal forces equivalent to an axial force and a couple
- *Transverse Loading*: Concentrated or distributed transverse load produces internal forces equivalent to a shear force and a couple
- *Principle of Superposition*: The normal stress due to pure bending may be combined with the normal stress due to axial loading and shear stress due to shear loading to find the complete state of stress.

Symmetric Member in Pure Bending



- Internal forces in any cross section are equivalent to a couple. The moment of the couple is the section *bending moment*.
- From statics, a couple M consists of equal and opposite forces.
- The sum of the components of the forces in any direction is zero.
- The moment is the same about any axis perpendicular to the plane of the couple and zero about any axis contained in the plane.
- These requirements may be applied to the sums of the components and moments of the statically indeterminate elementary internal forces.

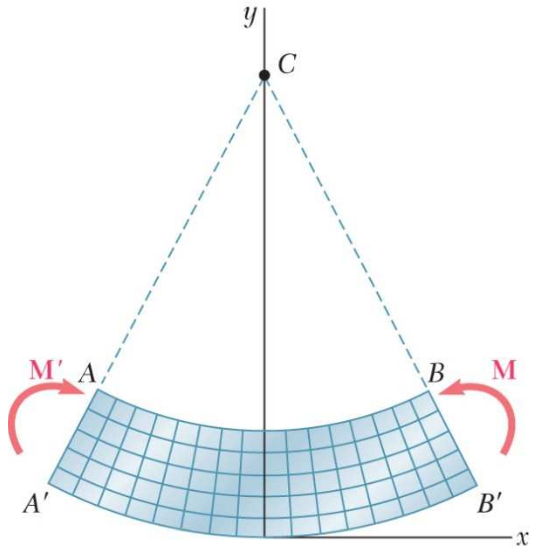


$$F_x = \int \sigma_x \, dA = 0$$

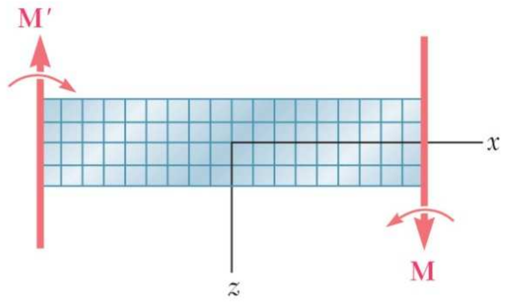
$$M_y = \int z\sigma_x \, dA = 0$$

$$M_z = \int -y\sigma_x \, dA = M$$

Bending Deformations



(*a*) Longitudinal, vertical section (plane of symmetry)



(*b*) Longitudinal, horizontal section

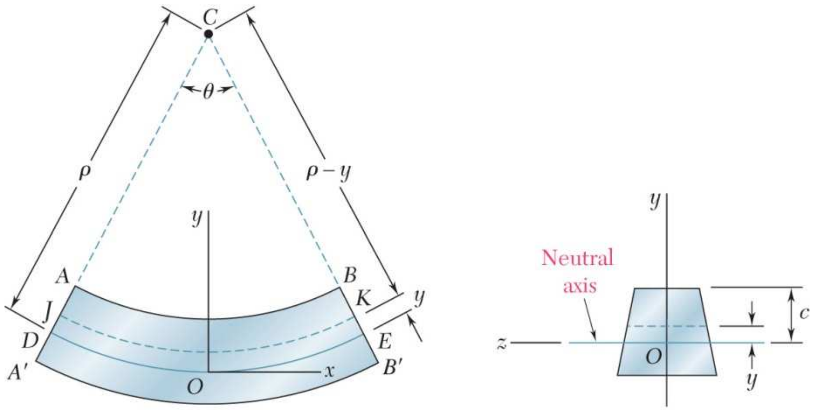
Beam with a plane of symmetry in pure bending:

- member remains symmetric
- bends uniformly to form a circular arc
- cross-sectional plane passes through arc center and remains planar
- length of top decreases and length of bottom increases
- a *neutral surface* must exist that is parallel to the upper and lower surfaces and for which the length does not change
- stresses and strains are negative (compressive) above the neutral plane and positive (tension) below it

Strain Due to Bending

Consider a beam segment of length L.

After deformation, the length of the neutral surface remains L. At other sections,



(*a*) Longitudinal, vertical section (plane of symmetry)

(*b*) Transverse section

$$L' = (\rho - y)\theta$$

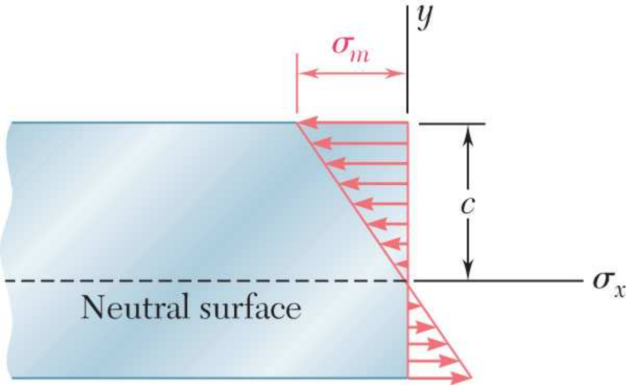
$$\delta = L' - L = (\rho - y)\theta - \rho\theta = -y\theta$$

$$\varepsilon_x = \frac{\delta}{L} = -\frac{y\theta}{\rho\theta} = -\frac{y}{\rho} \quad \text{(strain varies linearly)}$$

y is measured from the neutral surface

ρ is radius of curvature of neutral surface

Stress Due to Bending



- For a linearly elastic material,

$$\sigma_x = E\varepsilon_x = -E\frac{y}{\rho}$$

(stress varies linearly)

- For static equilibrium,

$$F_x = 0 = \int \sigma_x \, dA = \int -E\frac{y}{\rho} \, dA$$

$$0 = -\frac{E}{\rho}\int y \, dA$$

First moment with respect to neutral plane is zero. Therefore, the neutral surface must pass through the section centroid.

- For static equilibrium,

$$M = \int(-y\sigma_x \, dA) = \int(-y)\left(-E\frac{y}{\rho}\right) dA$$

$$M = \frac{E}{\rho}\int y^2 \, dA = \frac{EI_z}{\rho}$$

$$\frac{1}{\rho} = \frac{M}{EI_z}$$

Substituting into $\sigma_x = -E\frac{y}{\rho}$

$$\sigma_x = -\frac{My}{I_z}$$

Symmetry of Cross-Section

- For static equilibrium,

$$0 = \int (z\sigma_x \, dA) = \int (z)\left(-\frac{My}{I_z}\right) dA$$

$$0 = \frac{M}{I_z}\int yz \, dA$$

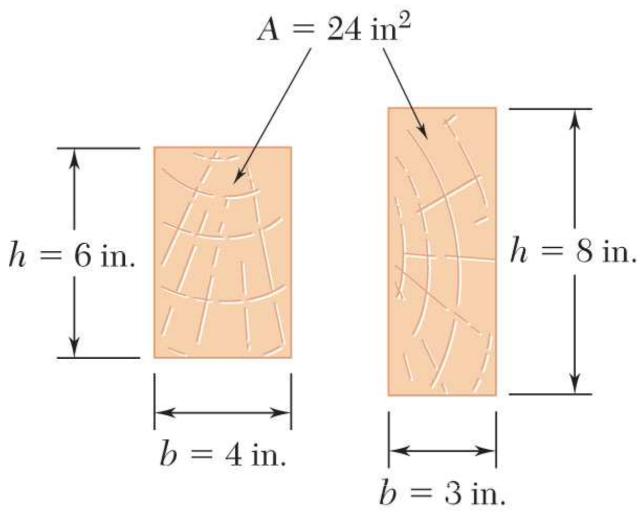
- The integral is zero if the cross-section is symmetric about either the y- or z-axis
- The flexure formula:

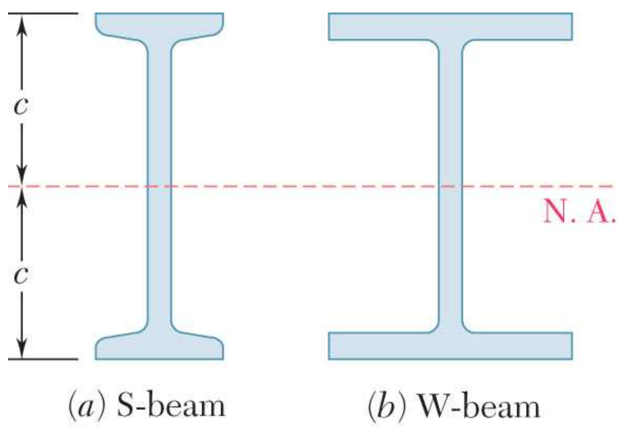
$$\sigma_x = -\frac{My}{I_z}$$

is limited to:

- Linear elastic material
- Bending about an axis of symmetry of the cross-section, **or** about an axis perpendicular to an axis of symmetry

Beam Section Properties





- The maximum normal stress due to bending,

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

$I =$ section moment of inertia

$$S = \frac{I}{c} = \text{section modulus}$$

A beam section with a larger section modulus will have a lower maximum stress

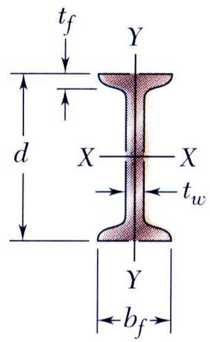
- Consider a rectangular beam cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^3 = \frac{1}{6}Ah$$

Between two beams with the same cross sectional area, the beam with the greater depth will be more effective in resisting bending.

- Structural steel beams are designed to have a large section modulus.

Properties of American Standard Shapes

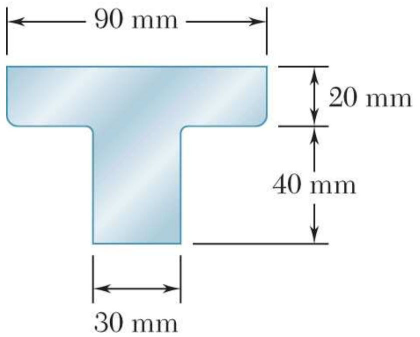


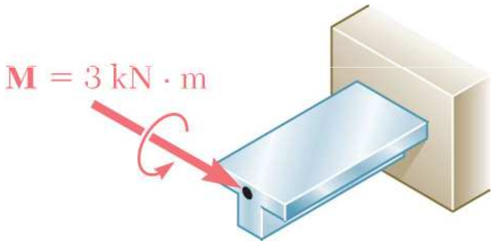
Appendix C. **Properties of Rolled-Steel Shapes**
(SI Units)

S Shapes
(American Standard Shapes)

Designation†	Area A, mm^2	Depth d, mm	Flange		Web Thickness t_w, mm	Axis X-X			Axis Y-Y		
			Width b_f, mm	Thickness t_f, mm		I_x 10^6 mm^4	S_x 10^3 mm^3	r_x mm	I_y 10^6 mm^4	S_y 10^3 mm^3	r_y mm
S610 × 180	22900	622	204	27.7	20.3	1320	4240	240	34.9	341	39.0
158	20100	622	200	27.7	15.7	1230	3950	247	32.5	321	39.9
149	19000	610	184	22.1	18.9	995	3260	229	20.2	215	32.3
134	17100	610	181	22.1	15.9	938	3080	234	19.0	206	33.0
119	15200	610	178	22.1	12.7	878	2880	240	17.9	198	34.0
S510 × 143	18200	516	183	23.4	20.3	700	2710	196	21.3	228	33.9
128	16400	516	179	23.4	16.8	658	2550	200	19.7	216	34.4
112	14200	508	162	20.2	16.1	530	2090	193	12.6	152	29.5
98.3	12500	508	159	20.2	12.8	495	1950	199	11.8	145	30.4
S460 × 104	13300	457	159	17.6	18.1	385	1685	170	10.4	127	27.5
81.4	10400	457	152	17.6	11.7	333	1460	179	8.83	113	28.8
S380 × 74	9500	381	143	15.6	14.0	201	1060	145	6.65	90.8	26.1
64	8150	381	140	15.8	10.4	185	971	151	6.15	85.7	27.1

Sample Problem 4.2





A cast-iron machine part is acted upon by a 3 kN-m couple. Knowing *E* = 165 GPa and neglecting the effects of fillets, determine (a) the maximum tensile and compressive stresses, (b) the radius of curvature.

SOLUTION:

- Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} \qquad I_{x'} = \sum\left(\bar{I} + Ad^2\right)$$

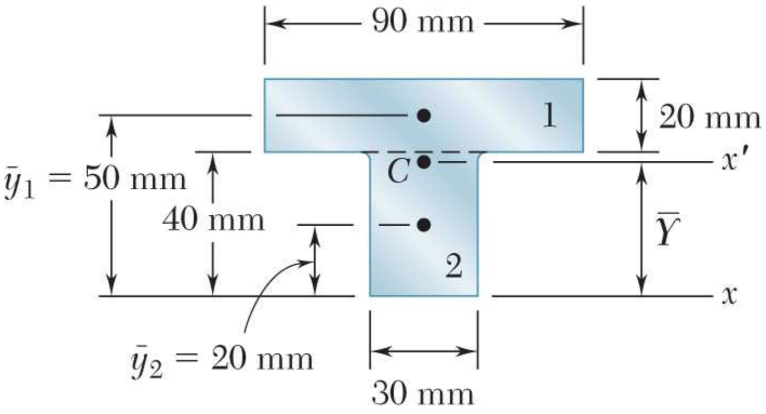
- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

- Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$

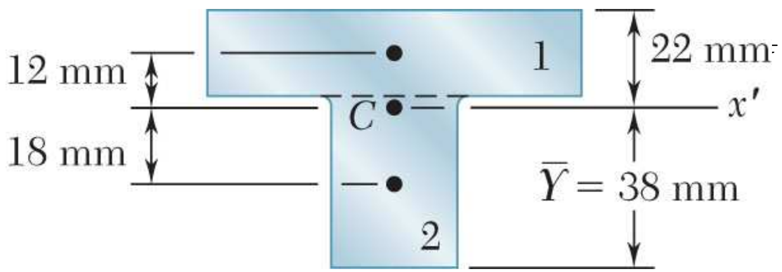
Sample Problem 4.2



SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$20\times90=1800$	50	90×10^3
2	$40\times30=1200$	20	24×10^3
	$\sum A = 3000$		$\sum \bar{y}A = 114\times10^3$



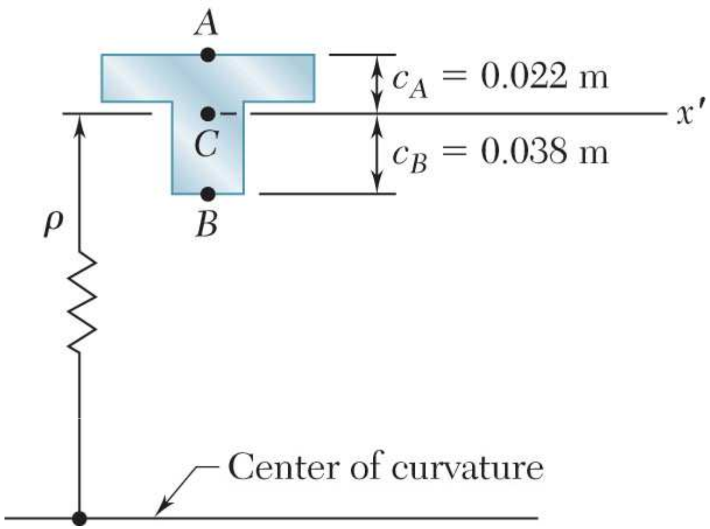
$$= \frac{\sum \bar{y}A}{\sum A} = \frac{114\times10^3}{3000} = 38\text{ mm}$$

$$I_{x'} = \sum\left(\bar{I} + Ad^2\right) = \sum\left(\tfrac{1}{12}bh^3 + Ad^2\right)$$

$$= \left(\tfrac{1}{12}90\times20^3 + 1800\times12^2\right) + \left(\tfrac{1}{12}30\times40^3 + 1200\times18^2\right)$$

$$I = 868\times10^3\text{ mm}^4 = 868\times10^{-9}\text{ m}^4$$

Sample Problem 4.2



- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_A = \frac{M\,c_A}{I} = \frac{3\,\text{kN}\cdot\text{m}\times 0.022\,\text{m}}{868\times10^{-9}\,\text{m}^4}$$

$$\boxed{\sigma_A = +76.0\,\text{MPa}}$$

$$\sigma_B = -\frac{M\,c_B}{I} = -\frac{3\,\text{kN}\cdot\text{m}\times 0.038\,\text{m}}{868\times10^{-9}\,\text{m}^4}$$

$$\boxed{\sigma_B = -131.3\,\text{MPa}}$$

- Calculate the curvature

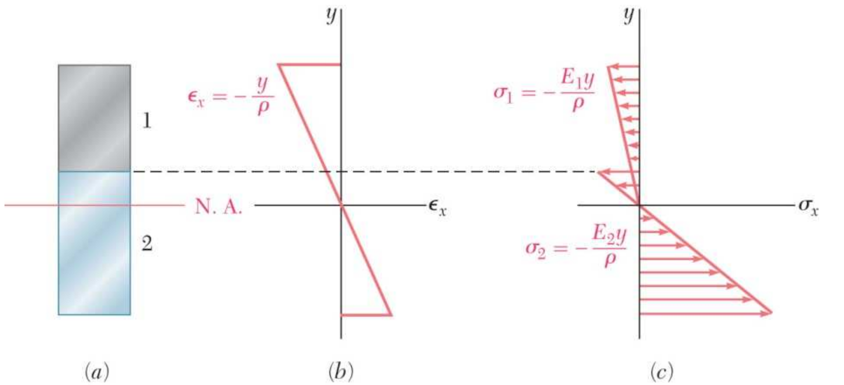
$$\frac{1}{\rho} = \frac{M}{EI}$$

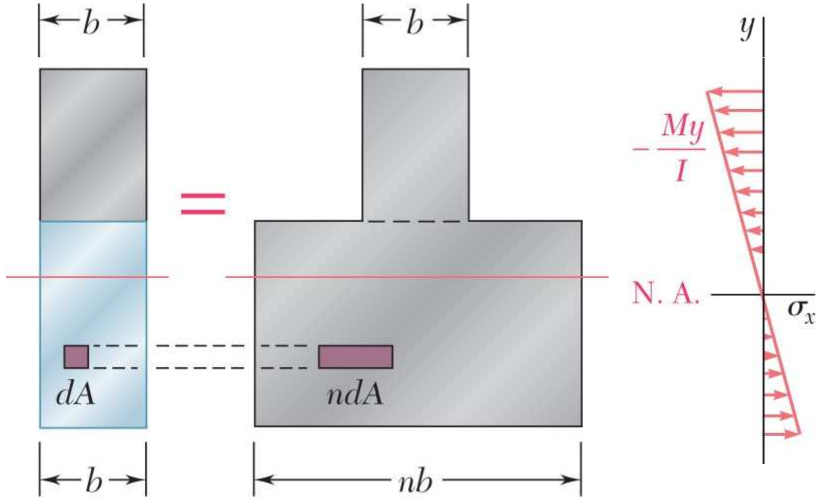
$$= \frac{3\,\text{kN}\cdot\text{m}}{(165\,\text{GPa})(868\times10^{-9}\,\text{m}^4)}$$

$$\frac{1}{\rho} = 20.95\times10^{-3}\,\text{m}^{-1}$$

$$\rho = 47.7\,\text{m}$$

Bending of Members Made of Several Materials





$$\sigma_x = -\frac{My}{I}$$

$$\sigma_1 = \sigma_x \qquad \sigma_2 = n\sigma_x$$

- Consider a composite beam formed from two materials with E_1 and E_2.
- Normal strain varies linearly.

$$\varepsilon_x = -\frac{y}{\rho}$$

- Piecewise linear normal stress variation.

$$\sigma_1 = E_1\varepsilon_x = -\frac{E_1 y}{\rho} \qquad \sigma_2 = E_2\varepsilon_x = -\frac{E_2 y}{\rho}$$

  Neutral axis does not pass through section centroid of composite section.

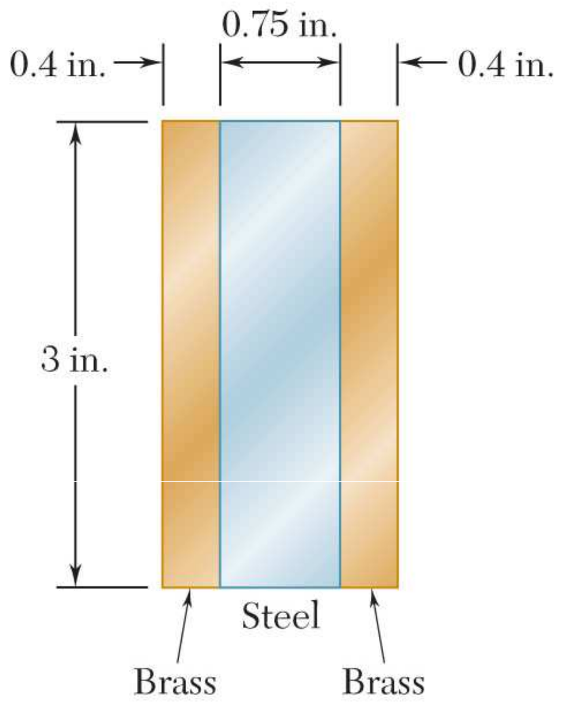
- Elemental forces on the section are

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA \quad dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA$$

- Define a transformed section such that

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(n\,dA) \qquad n = \frac{E_2}{E_1}$$

Example 4.03

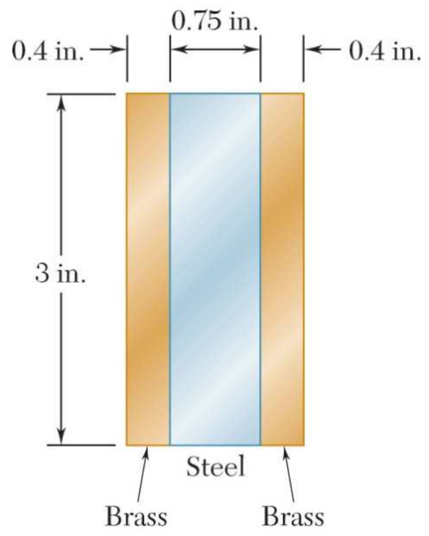


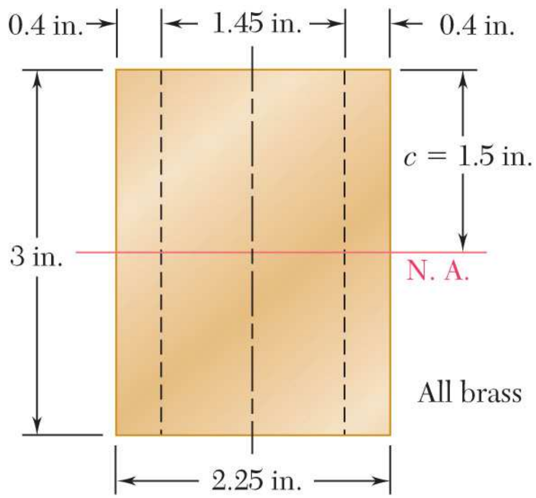
Bar is made from bonded pieces of steel (E_s = 29x10^6 psi) and brass (E_b = 15x10^6 psi). Determine the maximum stress in the steel and brass when a moment of 40 kip*in is applied.

SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass
- Evaluate the cross sectional properties of the transformed section
- Calculate the maximum stress in the transformed section. This is the correct maximum stress for the brass pieces of the bar.
- Determine the maximum stress in the steel portion of the bar by multiplying the maximum stress for the transformed section by the ratio of the moduli of elasticity.

Example 4.03





SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass.

$$n = \frac{E_s}{E_b} = \frac{29\times10^6\,\text{psi}}{15\times10^6\,\text{psi}} = 1.933$$

$$b_T = 0.4\,\text{in} + 1.933\times0.75\,\text{in} + 0.4\,\text{in} = 2.25\,\text{in}$$

- Evaluate the transformed cross sectional properties

$$I = \tfrac{1}{12} b_T h^3 = \tfrac{1}{12}(2.25\,\text{in.})(3\,\text{in.})^3$$

$$= 5.063\,\text{in.}^4$$

- Calculate the maximum stresses

$$\sigma_m = \frac{Mc}{I} = \frac{(40\,\text{kip}\cdot\text{in.})(1.5\,\text{in.})}{5.063\,\text{in.}^4} = 11.85\,\text{ksi}$$

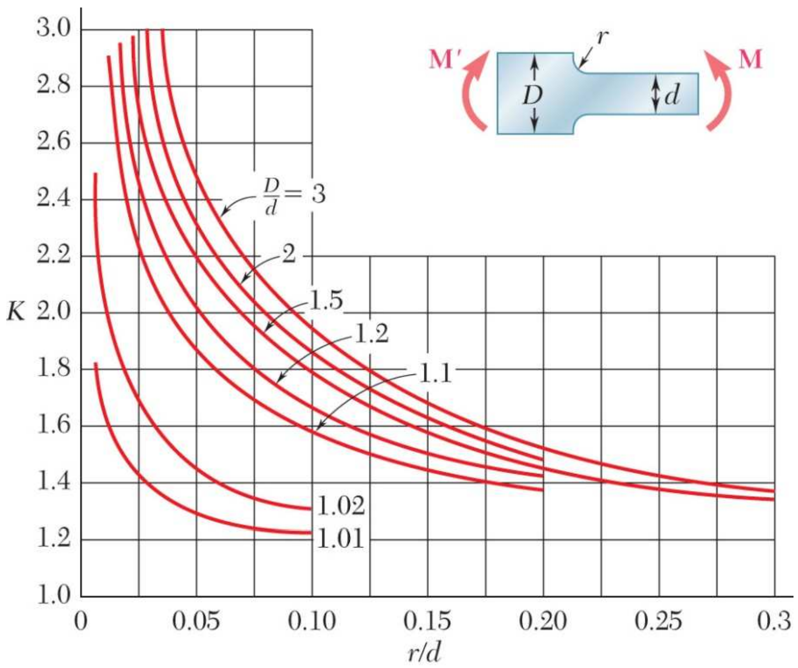
$$(\sigma_b)_{\max} = \sigma_m$$

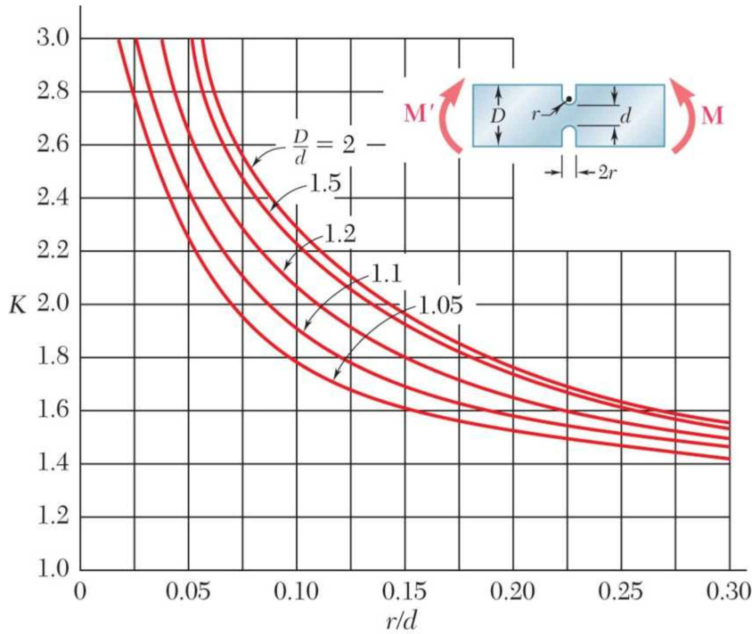
$$(\sigma_s)_{\max} = n\sigma_m = 1.933\times11.85\,\text{ksi}$$

$$(\sigma_b)_{\max} = 11.85\,\text{ksi}$$

$$(\sigma_s)_{\max} = 22.9\,\text{ksi}$$

Stress Concentrations



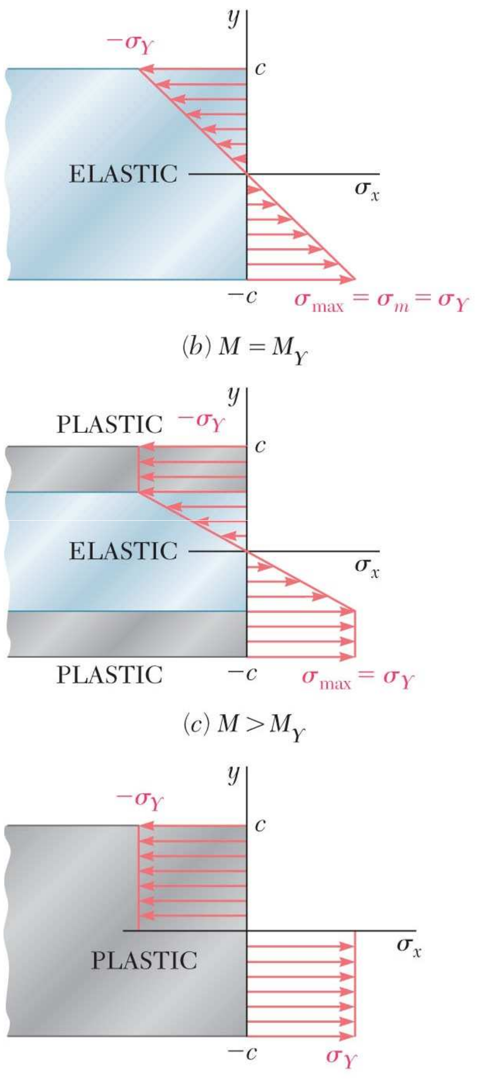


Stress concentrations may occur:

- in the vicinity of points where the loads are applied
- in the vicinity of abrupt changes in cross section

$$\sigma_m = K\frac{Mc}{I}$$

Members Made of an Elastoplastic Material



(b) $M = M_Y$

(c) $M > M_Y$

(d) $M = M_p$

- Rectangular beam made of an elastoplastic material

$$\sigma_x \le \sigma_Y \qquad \sigma_m = \frac{Mc}{I}$$

$$\sigma_m = \sigma_Y \qquad M_Y = \frac{I}{c}\sigma_Y = \text{maximum elastic moment}$$

- If the moment is increased beyond the maximum elastic moment, plastic zones develop around an elastic core.

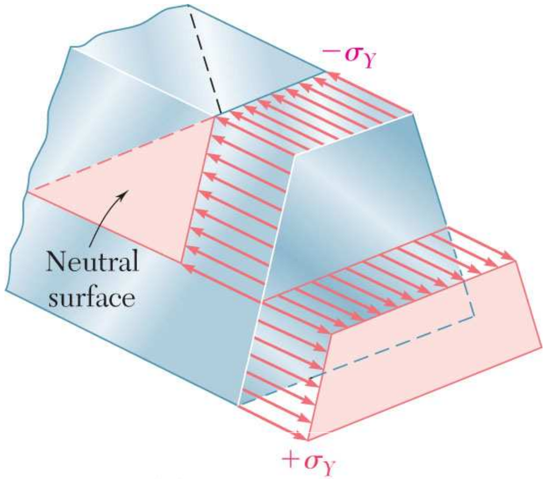
$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \qquad y_Y = \text{elastic core half - thickness}$$

- In the limit as the moment is increased further, the elastic core thickness goes to zero, corresponding to a fully plastic deformation.

$$M_p = \frac{3}{2}M_Y = \text{plastic moment}$$

$$k = \frac{M_p}{M_Y} = \text{shape factor (depends only on cross section shape)}$$

Plastic Deformations of Members With a Single Plane of Symmetry



(a)

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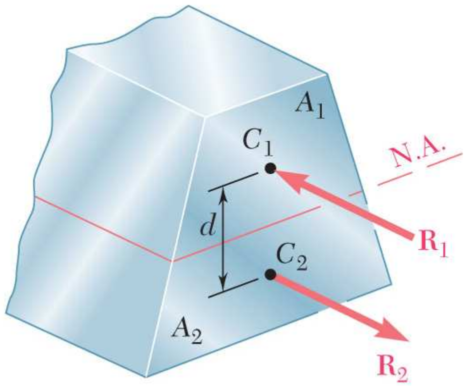


(b)

- Fully plastic deformation of a beam with only a vertical plane of symmetry.
- The neutral axis cannot be assumed to pass through the section centroid.
- Resultants R_1 and R_2 of the elementary compressive and tensile forces form a couple.

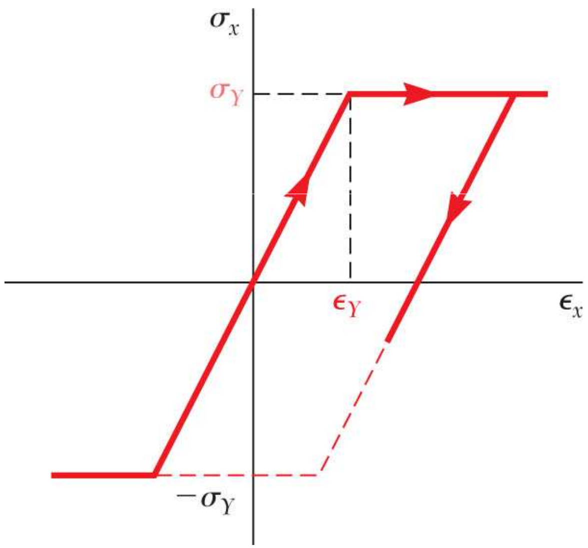
$$R_1 = R_2$$
$$A_1\sigma_Y = A_2\sigma_Y$$

The neutral axis divides the section into equal areas.

- The plastic moment for the member,

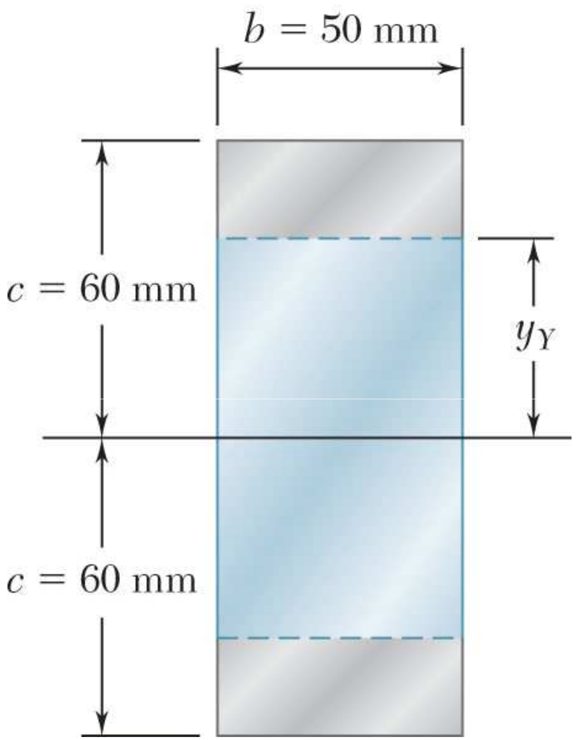
$$M_p = \left(\tfrac{1}{2}A\sigma_Y\right)d$$

Residual Stresses



- Plastic zones develop in a member made of an elastoplastic material if the bending moment is large enough.
- Since the linear relation between normal stress and strain applies at all points during the unloading phase, it may be handled by assuming the member to be fully elastic.
- Residual stresses are obtained by applying the principle of superposition to combine the stresses due to loading with a moment *M* (elastoplastic deformation) and unloading with a moment -*M* (elastic deformation).
- The final value of stress at a point will not, in general, be zero.

Example 4.05, 4.06

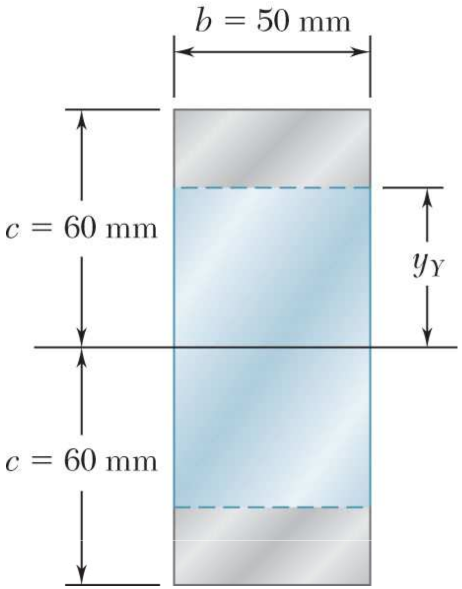


A member of uniform rectangular cross section is subjected to a bending moment M = 36.8 kN-m. The member is made of an elastoplastic material with a yield strength of 240 MPa and a modulus of elasticity of 200 GPa.

Determine (a) the thickness of the elastic core, (b) the radius of curvature of the neutral surface.

After the loading has been reduced back to zero, determine (c) the distribution of residual stresses, (d) radius of curvature.

Example 4.05, 4.06



- Maximum elastic moment:

$$\frac{I}{c} = \tfrac{2}{3}bc^2 = \tfrac{2}{3}\left(50\times10^{-3}\,\text{m}\right)\left(60\times10^{-3}\,\text{m}\right)^2$$

$$= 120\times10^{-6}\,\text{m}^3$$

$$M_Y = \frac{I}{c}\sigma_Y = \left(120\times10^{-6}\,\text{m}^3\right)\left(240\,\text{MPa}\right)$$

$$= 28.8\,\text{kN}\cdot\text{m}$$

- Thickness of elastic core:

$$M = \tfrac{3}{2}M_Y\left(1-\tfrac{1}{3}\frac{y_Y^2}{c^2}\right)$$

$$36.8\,\text{kN}\cdot\text{m} = \tfrac{3}{2}\left(28.8\,\text{kN}\cdot\text{m}\right)\left(1-\tfrac{1}{3}\frac{y_Y^2}{c^2}\right)$$

$$\frac{y_Y}{c} = \frac{y_Y}{60\,\text{mm}} = 0.666$$

$$2y_Y = 80\,\text{mm}$$

- Radius of curvature:

$$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{240\times10^6\,\text{Pa}}{200\times10^9\,\text{Pa}}$$

$$= 1.2\times10^{-3}$$

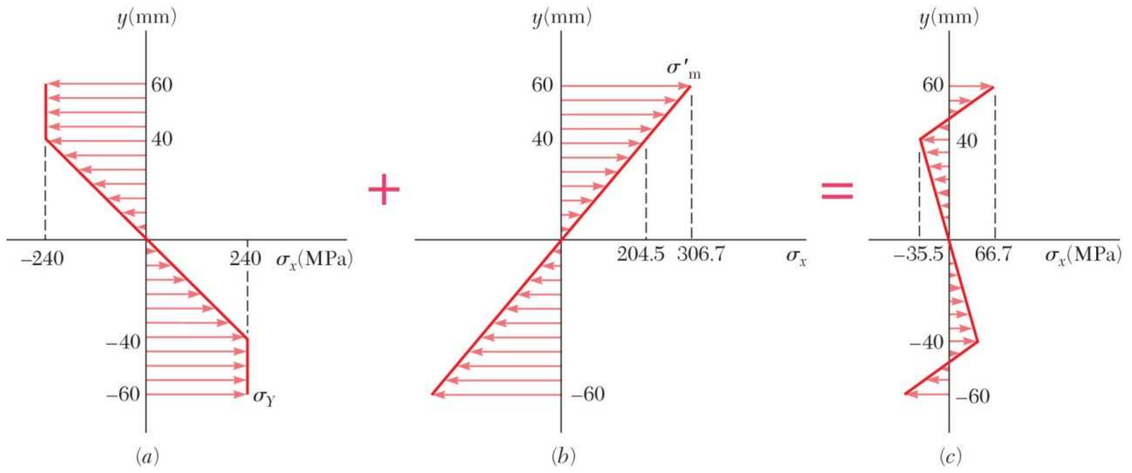
$$\varepsilon_Y = \frac{y_Y}{\rho}$$

$$\rho = \frac{y_Y}{\varepsilon_Y} = \frac{40\times10^{-3}\,\text{m}}{1.2\times10^{-3}}$$

$$\rho = 33.3\,\text{m}$$

End

Example 4.05, 4.06

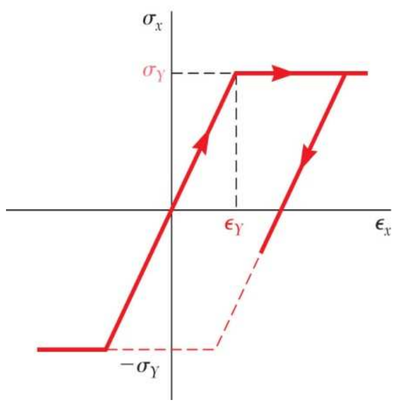


- M = 36.8 kN-m

$$y_Y = 40\,\text{mm}$$
$$\sigma_Y = 240\,\text{MPa}$$

- M = -36.8 kN-m

$$\sigma'_m = \frac{Mc}{I} = \frac{36.8\,\text{kN}\cdot\text{m}}{120\times10^6\,\text{m}^3}$$
$$= 306.7\,\text{MPa} < 2\sigma_Y$$



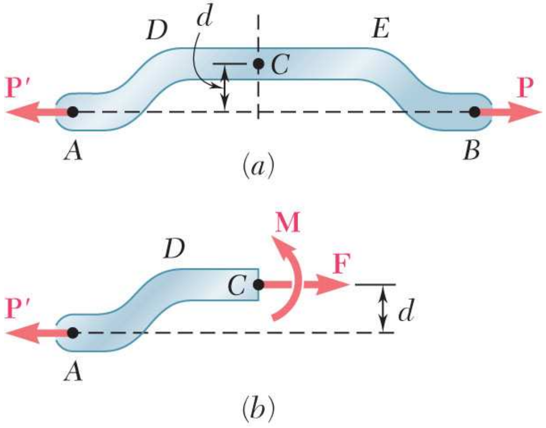
- $M = 0$

At the edge of the elastic core,

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{-35.5\times10^6\,\text{Pa}}{200\times10^9\,\text{Pa}}$$
$$= -177.5\times10^{-6}$$
$$\rho = -\frac{y_Y}{\varepsilon_x} = \frac{40\times10^{-3}\,\text{m}}{177.5\times10^{-6}}$$

$$\boxed{\rho = 225\,\text{m}}$$

Eccentric Axial Loading in a Plane of Symmetry



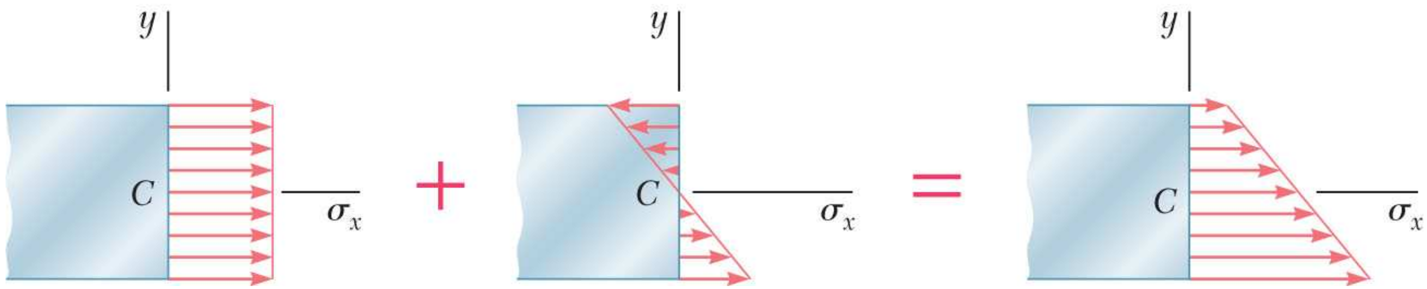
- Eccentric loading

$$F = P$$
$$M = Pd$$

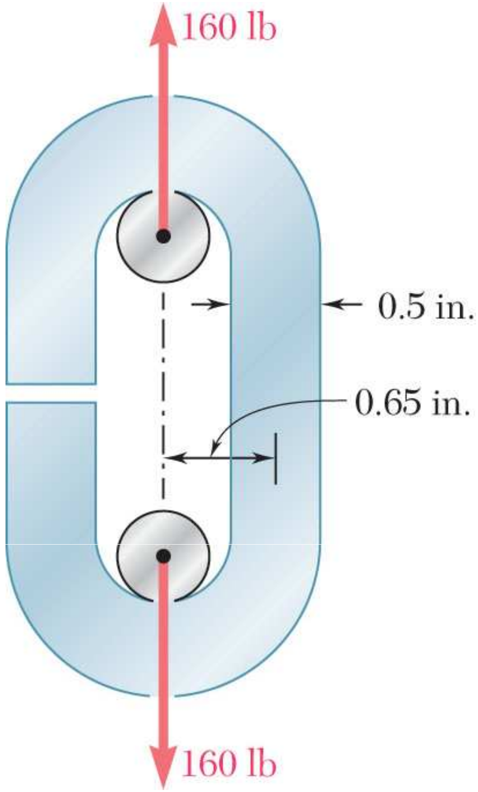
- Stress due to eccentric loading found by superposing the uniform stress due to a centric load and linear stress distribution due a pure bending moment

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$
$$= \frac{P}{A} - \frac{My}{I}$$

- Validity requires stresses below proportional limit, deformations have negligible effect on geometry, and stresses not evaluated near points of load application.



Example 4.07

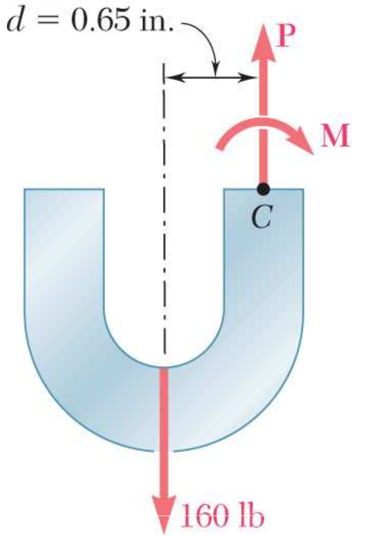


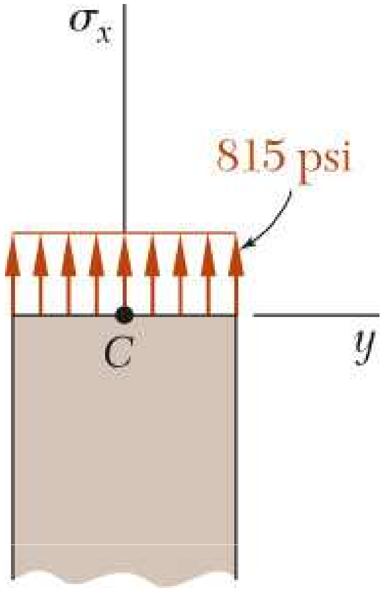
An open-link chain is obtained by bending low-carbon steel rods into the shape shown. For 160 lb load, determine (a) maximum tensile and compressive stresses, (b) distance between section centroid and neutral axis

SOLUTION:

- Find the equivalent centric load and bending moment
- Superpose the uniform stress due to the centric load and the linear stress due to the bending moment.
- Evaluate the maximum tensile and compressive stresses at the inner and outer edges, respectively, of the superposed stress distribution.
- Find the neutral axis by determining the location where the normal stress is zero.

Example 4.07



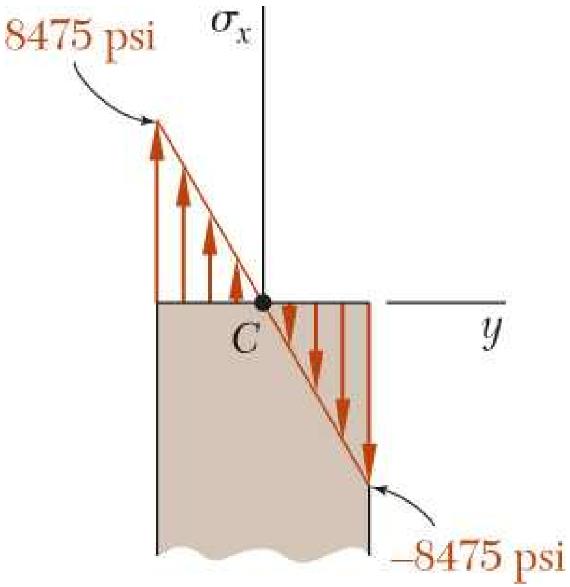


- Normal stress due to a centric load

$$A = \pi c^2 = \pi(0.25\,\text{in})^2$$
$$= 0.1963\,\text{in}^2$$
$$\sigma_0 = \frac{P}{A} = \frac{160\,\text{lb}}{0.1963\,\text{in}^2}$$
$$= 815\,\text{psi}$$

- Equivalent centric load and bending moment

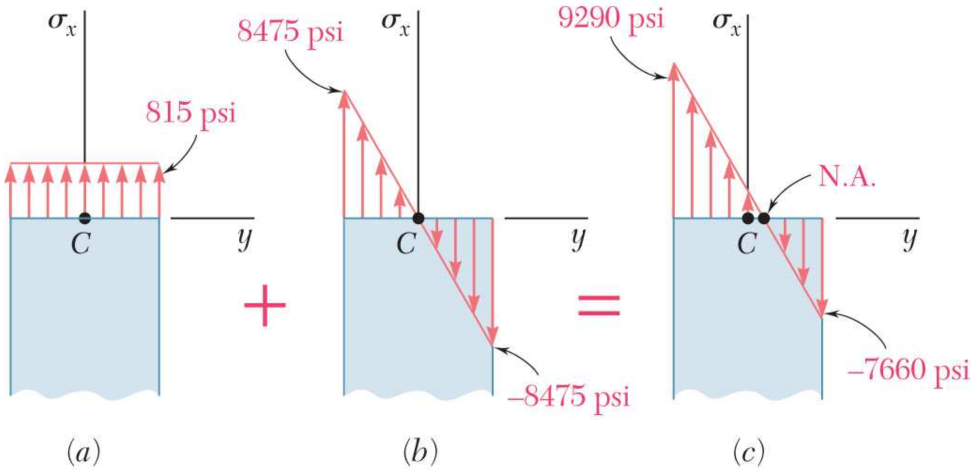
$$P = 160\,\text{lb}$$
$$M = Pd = (160\,\text{lb})(0.65\,\text{in})$$
$$= 104\,\text{lb}\cdot\text{in}$$



- Normal stress due to bending moment

$$I = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(0.25)^4$$
$$= 3.068\times10^{-3}\,\text{in}^4$$
$$\sigma_m = \frac{Mc}{I} = \frac{(104\,\text{lb}\cdot\text{in})(0.25\,\text{in})}{3.068\times10^{-3}\,\text{in}^4}$$
$$= 8475\,\text{psi}$$

Example 4.07



- Maximum tensile and compressive stresses

$$\sigma_t = \sigma_0 + \sigma_m = 815 + 8475$$

$$\boxed{\sigma_t = 9260\,\text{psi}}$$

$$\sigma_c = \sigma_0 - \sigma_m = 815 - 8475$$

$$\boxed{\sigma_c = -7660\,\text{psi}}$$

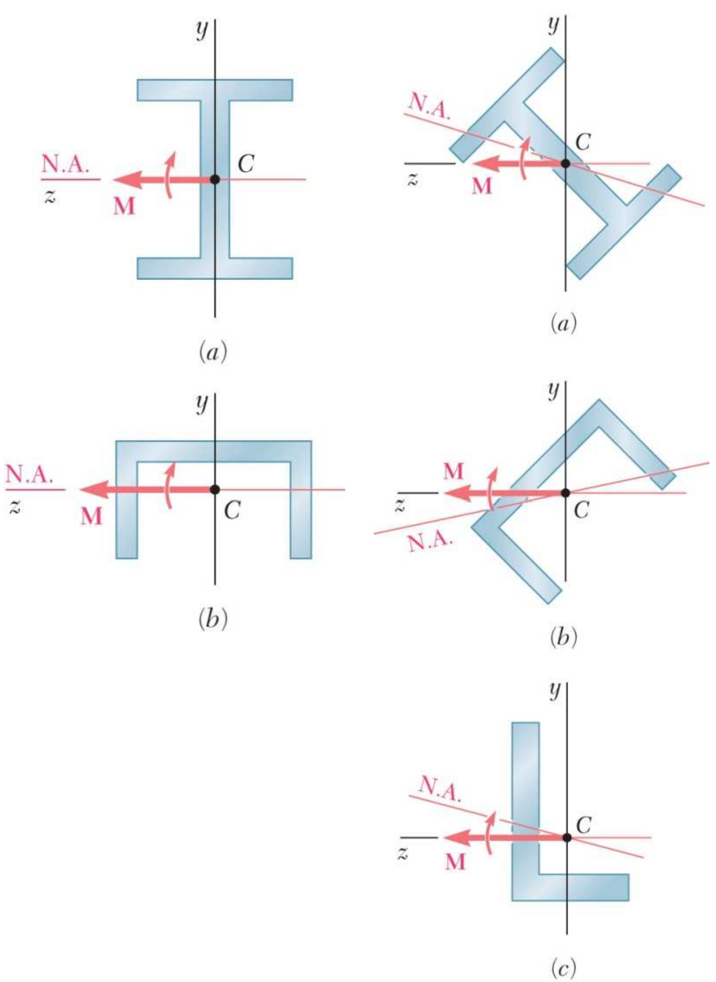
- Neutral axis location

$$0 = \frac{P}{A} - \frac{My_0}{I}$$

$$y_0 = \frac{P}{A}\frac{I}{M} = (815\,\text{psi})\frac{3.068\times10^{-3}\,\text{in}^4}{105\,\text{lb}\cdot\text{in}}$$

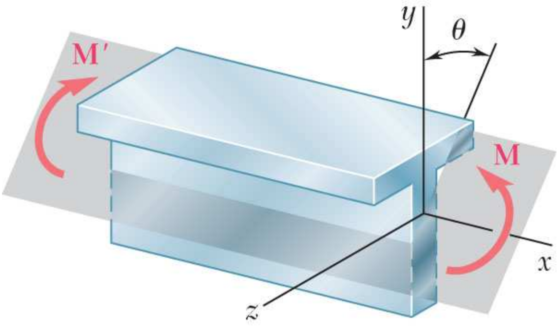
$$\boxed{y_0 = 0.0240\,\text{in}}$$

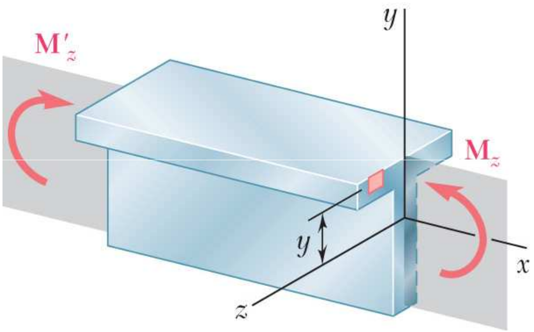
Unsymmetric Bending

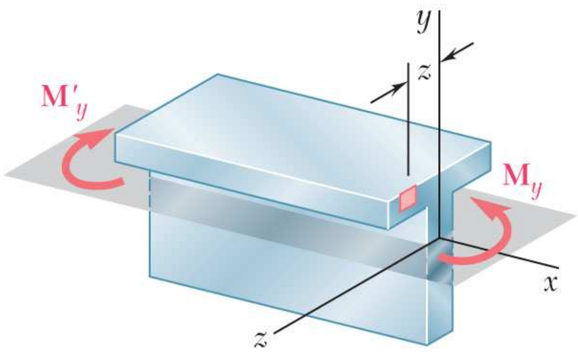


- Analysis of pure bending has been limited to members subjected to bending couples acting in a plane of symmetry.
- Members remain symmetric and bend in the plane of symmetry.
- The neutral axis of the cross section coincides with the axis of the couple.
- Will now consider situations in which the bending couples do not act in a plane of symmetry.
- Cannot assume that the member will bend in the plane of the couples.
- In general, the neutral axis of the section will not coincide with the axis of the couple.

Unsymmetric Bending







Superposition is applied to determine stresses in the most general case of unsymmetric bending.

- Resolve the couple vector into components along the principle centroidal axes.

$$M_z = M\cos\theta \qquad M_y = M\sin\theta$$

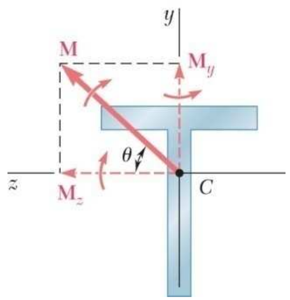
- Superpose the component stress distributions

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y y}{I_y}$$

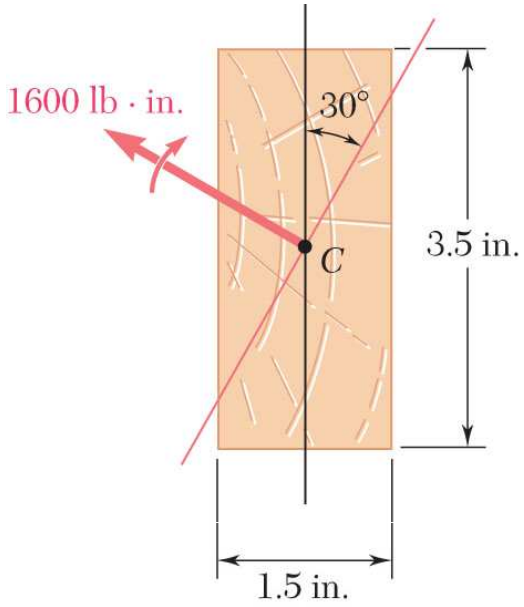
- Along the neutral axis,

$$\sigma_x = 0 = -\frac{M_z y}{I_z} + \frac{M_y y}{I_y} = -\frac{(M\cos\theta)y}{I_z} + \frac{(M\sin\theta)y}{I_y}$$

$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$



Example 4.08



A 1600 lb-in couple is applied to a rectangular wooden beam in a plane forming an angle of 30 deg. with the vertical. Determine (a) the maximum stress in the beam, (b) the angle that the neutral axis forms with the horizontal plane.

SOLUTION:

- Resolve the couple vector into components along the principle centroidal axes and calculate the corresponding maximum stresses.

$$M_z = M\cos\theta \qquad M_y = M\sin\theta$$

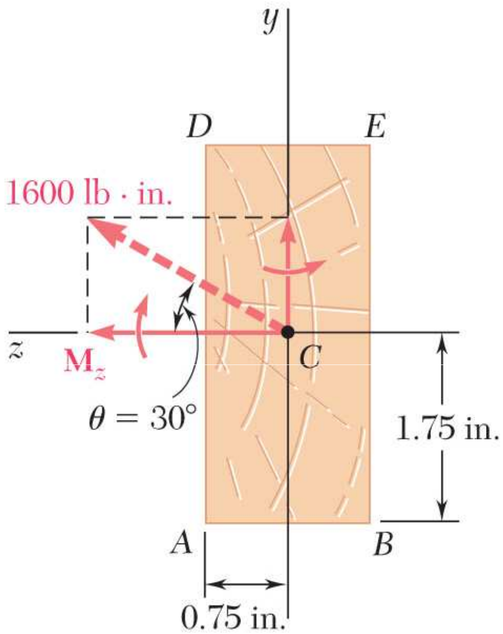
- Combine the stresses from the component stress distributions.

$$\sigma_x = \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- Determine the angle of the neutral axis.

$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$

Example 4.08



- Resolve the couple vector into components and calculate the corresponding maximum stresses.

$$M_z = (1600\,\text{lb}\cdot\text{in})\cos 30 = 1386\,\text{lb}\cdot\text{in}$$
$$M_y = (1600\,\text{lb}\cdot\text{in})\sin 30 = 800\,\text{lb}\cdot\text{in}$$
$$I_z = \tfrac{1}{12}(1.5\,\text{in})(3.5\,\text{in})^3 = 5.359\,\text{in}^4$$
$$I_y = \tfrac{1}{12}(3.5\,\text{in})(1.5\,\text{in})^3 = 0.9844\,\text{in}^4$$

The largest tensile stress due to M_z occurs along AB

$$\sigma_1 = \frac{M_z y}{I_z} = \frac{(1386\,\text{lb}\cdot\text{in})(1.75\,\text{in})}{5.359\,\text{in}^4} = 452.6\,\text{psi}$$

The largest tensile stress due to M_z occurs along AD

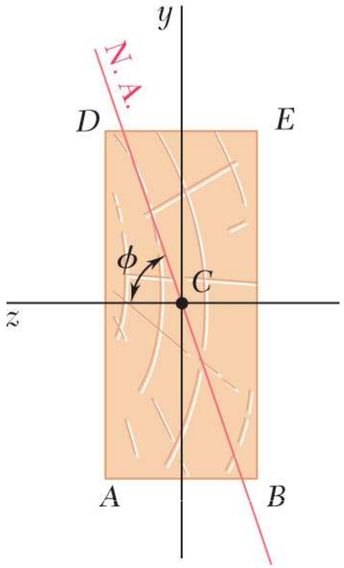
$$\sigma_2 = \frac{M_y z}{I_y} = \frac{(800\,\text{lb}\cdot\text{in})(0.75\,\text{in})}{0.9844\,\text{in}^4} = 609.5\,\text{psi}$$

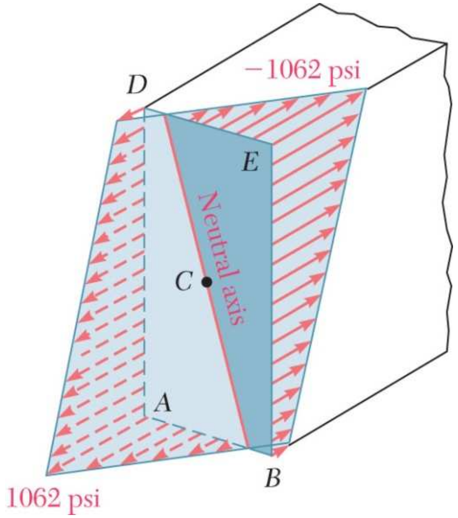
- The largest tensile stress due to the combined loading occurs at *A*.

$$\sigma_{max} = \sigma_1 + \sigma_2 = 452.6 + 609.5$$

$$\boxed{\sigma_{max} = 1062\,\text{psi}}$$

Example 4.08





- Determine the angle of the neutral axis.

$$\tan\phi = \frac{I_z}{I_y}\tan\theta = \frac{5.359\,\text{in}^4}{0.9844\,\text{in}^4}\tan 30$$

$$= 3.143$$

$$\boxed{\phi = 72.4^o}$$