Sixth Edition

CHAPTER

2

MECHANICS OF MATERIALS

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Stress and Strain – Axial Loading

Lecture Notes:
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Stress & Strain: Axial Loading

- Suitability of a structure or machine may depend on the deformations in the structure as well as the stresses induced under loading. Statics analyses alone are not sufficient.

- Considering structures as deformable allows determination of member forces and reactions which are statically indeterminate.

- Determination of the stress distribution within a member also requires consideration of deformations in the member.

- Chapter 2 is concerned with deformation of a structural member under axial loading. Later chapters will deal with torsional and pure bending loads.

End

Normal Strain

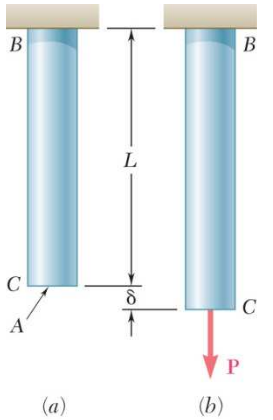


Fig. 2.1

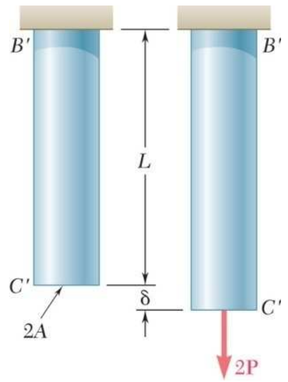


Fig. 2.3

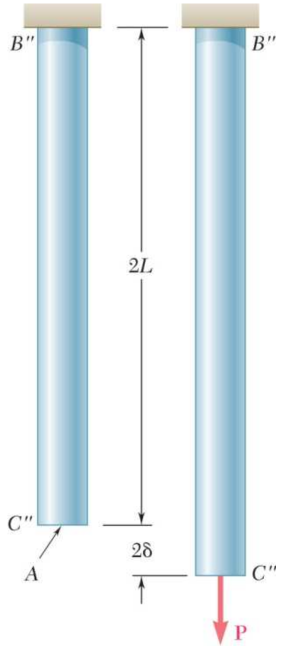


Fig. 2.4

$$\sigma = \frac{P}{A} = \text{stress}$$

$$\varepsilon = \frac{\delta}{L} = \text{normal strain}$$

$$\sigma = \frac{2P}{2A} = \frac{P}{A}$$

$$\varepsilon = \frac{\delta}{L}$$

$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Stress-Strain Test

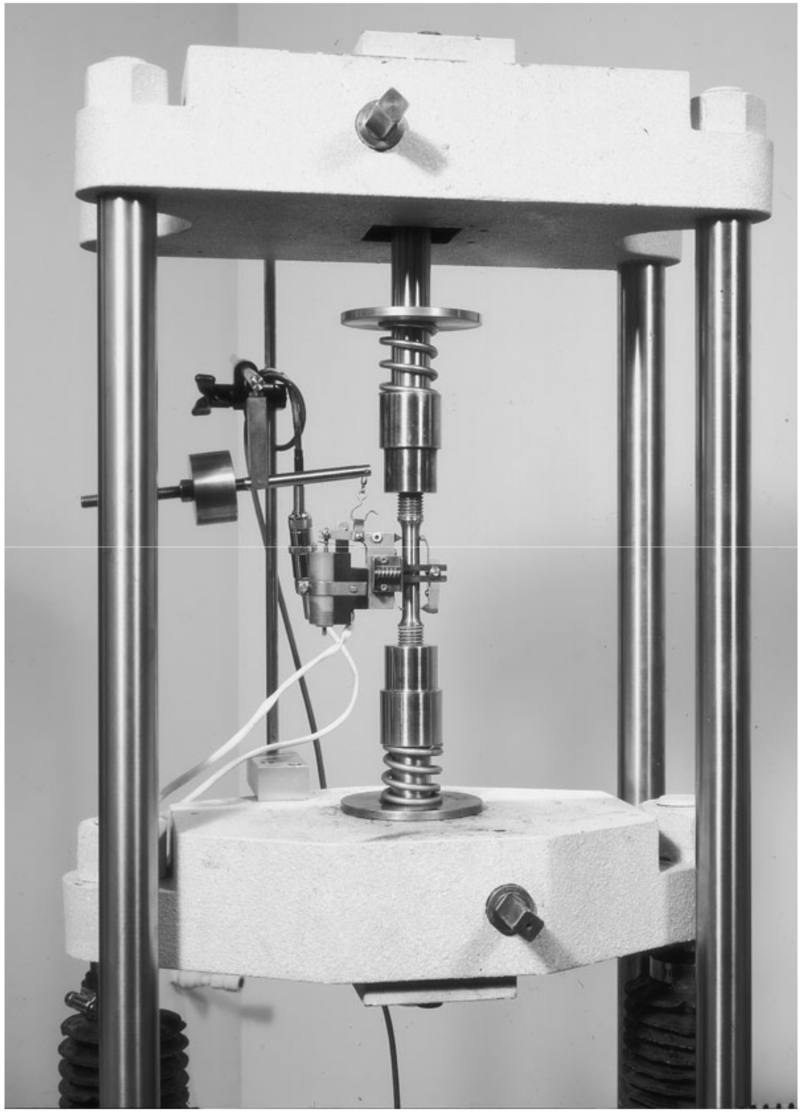


Fig 2.7 This machine is used to test tensile test specimens, such as those shown in this chapter.

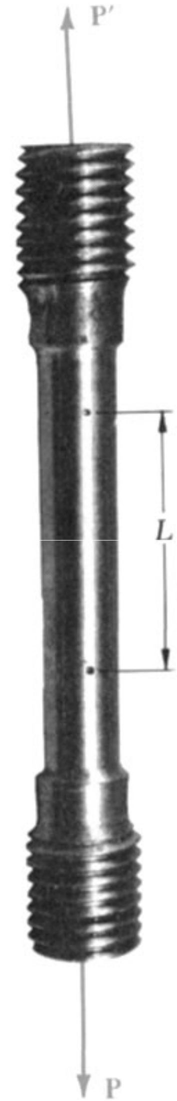
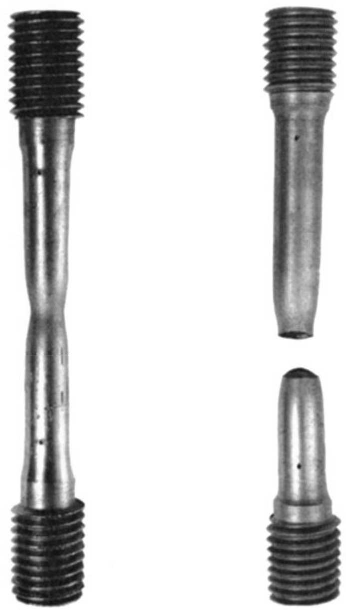
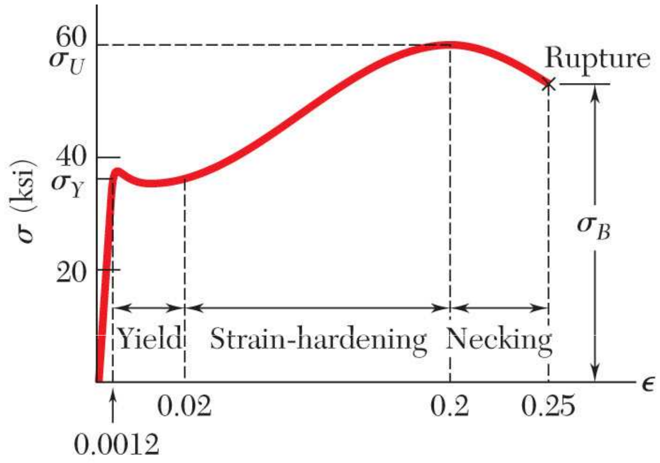


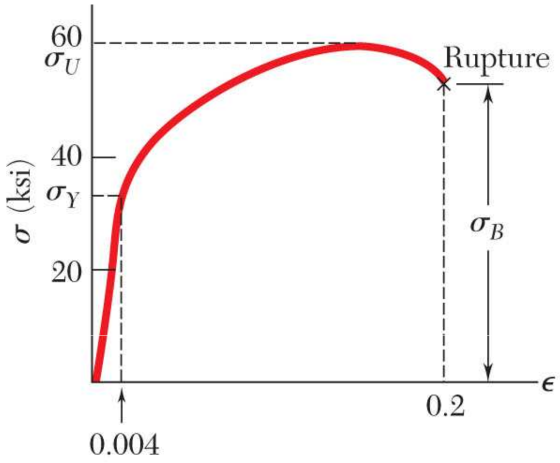
Fig 2.8 Test specimen with tensile load.

Stress-Strain Diagram: Ductile Materials


(a) (b)

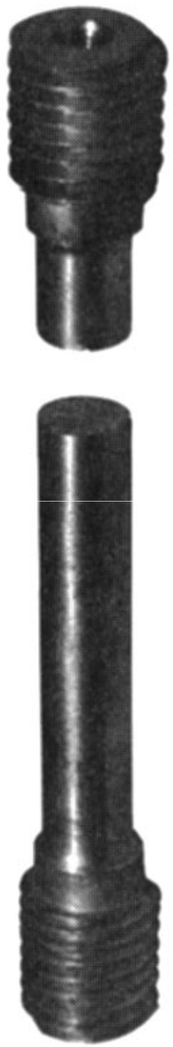


(a) Low-carbon steel



(b) Aluminum alloy

Stress-Strain Diagram: Brittle Materials



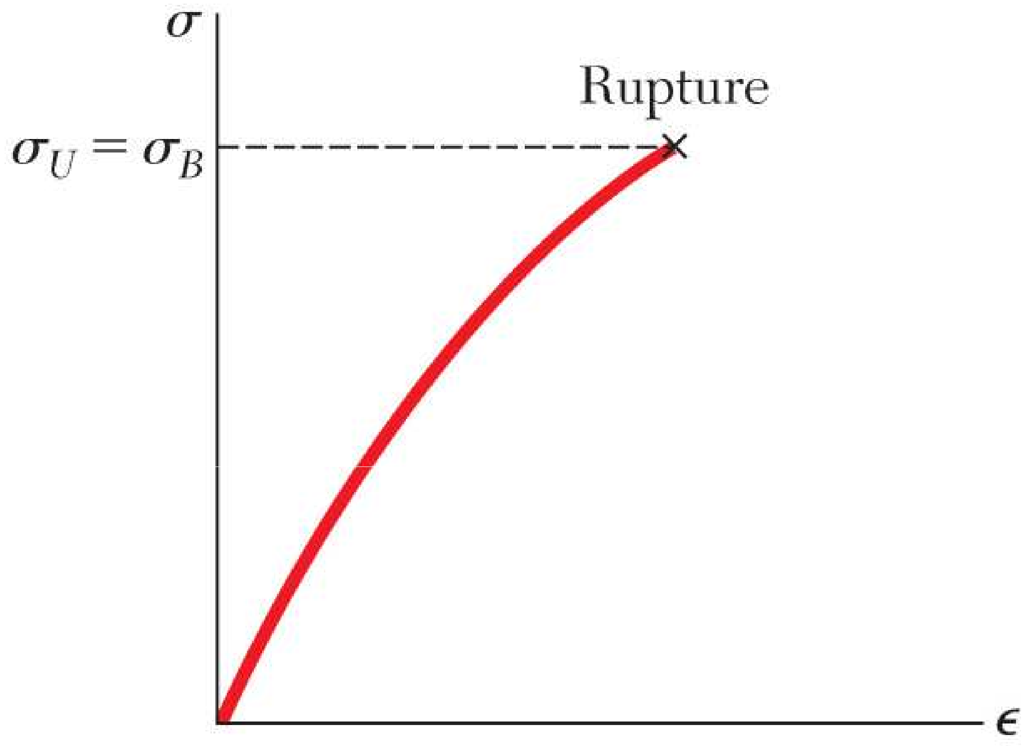


Fig 2.1 Stress-strain diagram for a typical brittle material.

Hooke's Law: Modulus of Elasticity

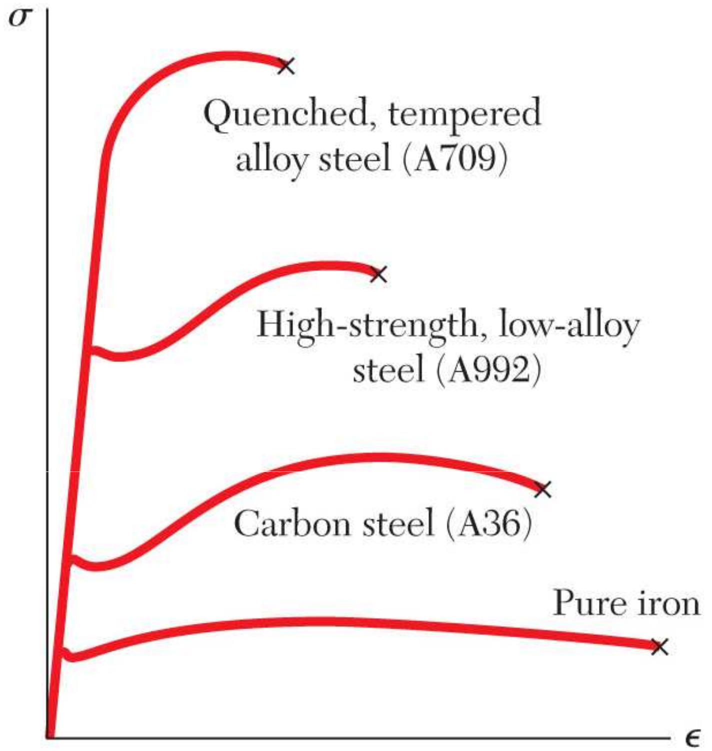


Fig 2.16 Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

$$\sigma = E\varepsilon$$

$$E = \text{Youngs Modulus or Modulus of Elasticity}$$

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

End

Elastic vs. Plastic Behavior

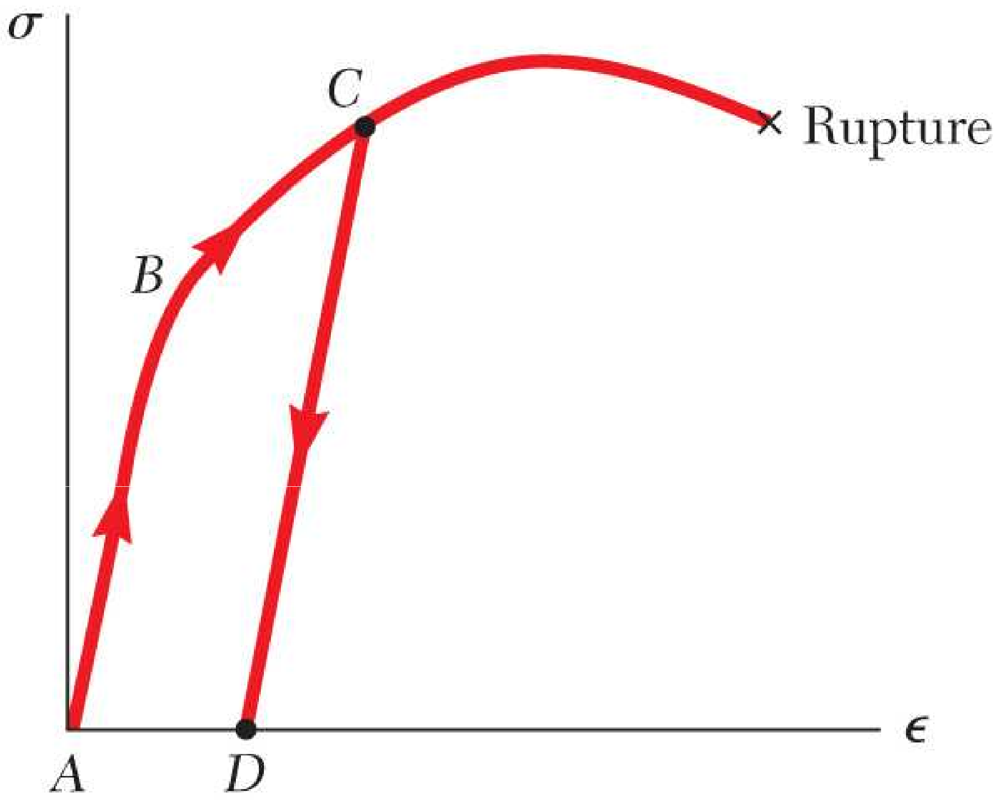


Fig. 2.18

- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

Fatigue

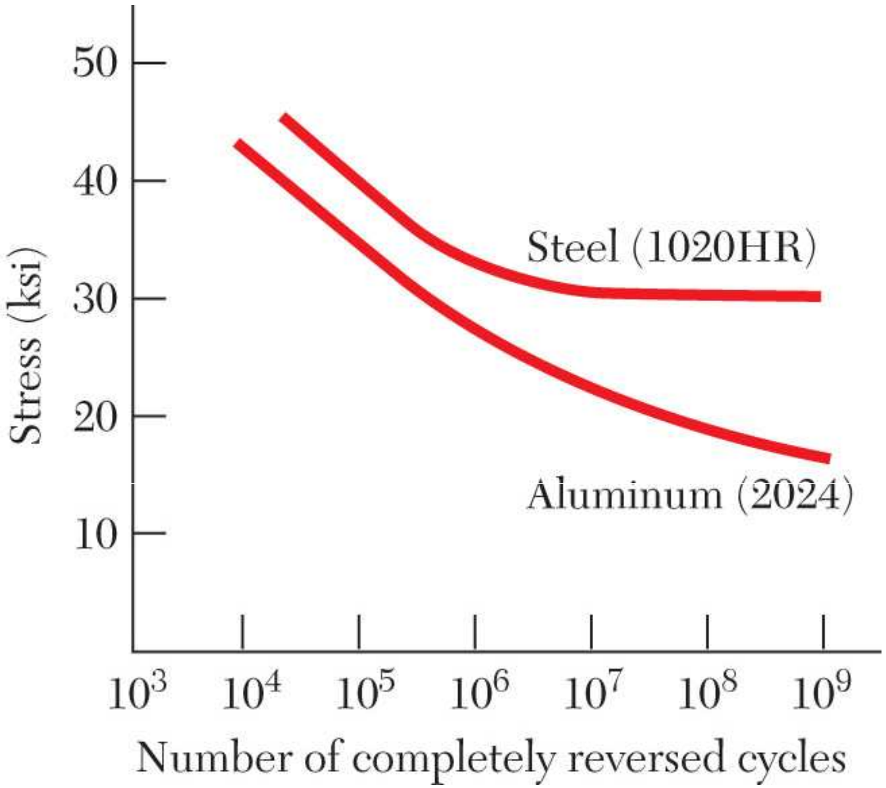


Fig. 2.21

- Fatigue properties are shown on S-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the *endurance limit*, fatigue failures do not occur for any number of cycles.

Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

$$FS = \text{Factor of safety}$$

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to integrity of whole structure
- risk to life and property
- influence on machine function

Deformations Under Axial Loading

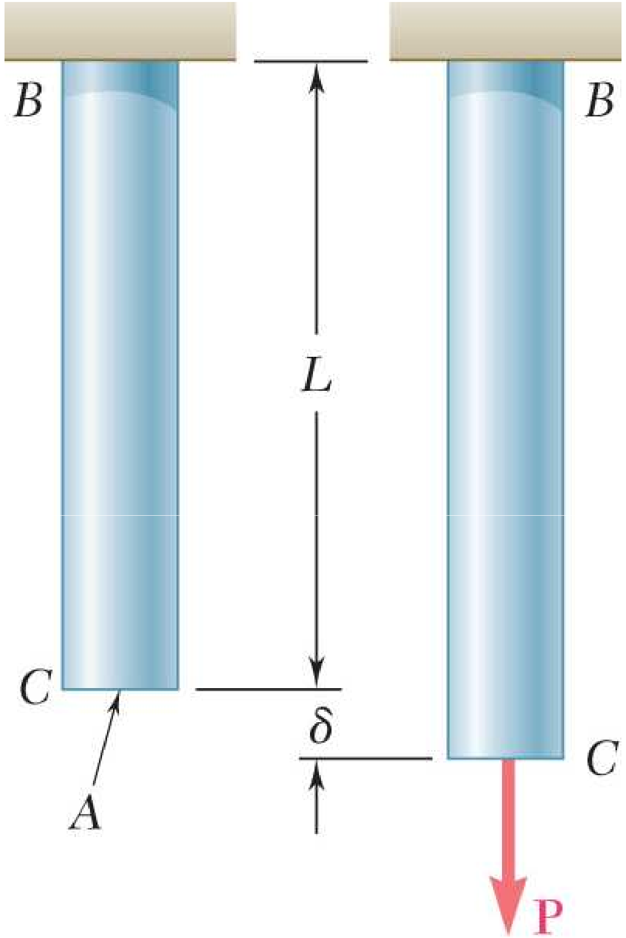


Fig. 2.22

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

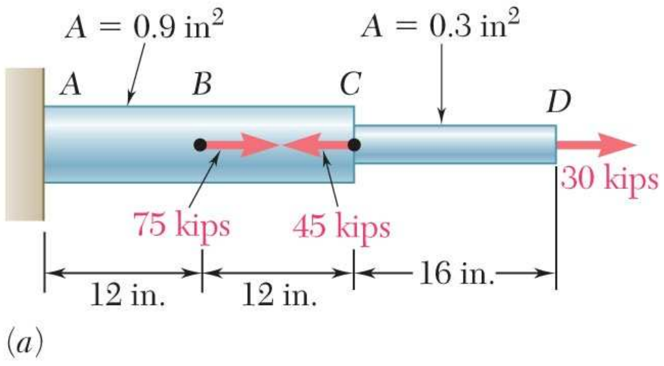
- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Example 2.01



$$E = 29\times10^{-6}\,\text{psi}$$

$$D = 1.07\text{ in.}\quad d = 0.618\text{ in.}$$

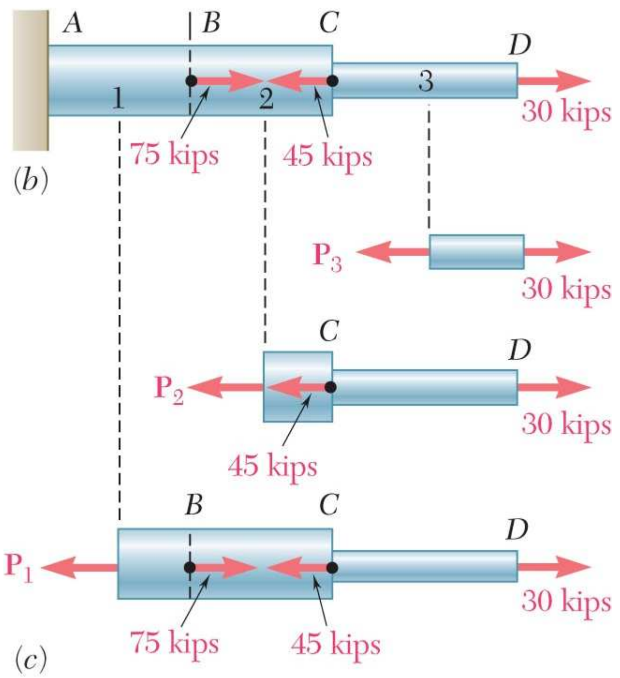
Determine the total deformation of the steel rod shown under the given loads.

SOLUTION:

- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

SOLUTION:

- Divide the rod into three components:



$L_1 = L_2 = 12\text{ in.}$ $\quad L_3 = 16\text{ in.}$

$A_1 = A_2 = 0.9\text{ in}^2$ $\quad A_3 = 0.3\text{ in}^2$

- Apply free-body analysis to each component to determine internal forces,

$$P_1 = 60\times10^3\text{lb}$$

$$P_2 = -15\times10^3\text{lb}$$

$$P_3 = 30\times10^3\text{lb}$$

- Evaluate total deflection,

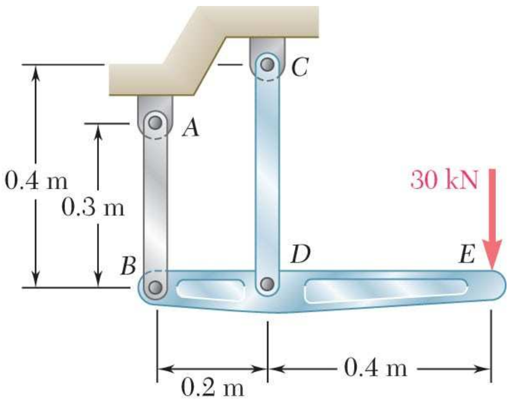
$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{29\times10^6}\left[\frac{\left(60\times10^3\right)12}{0.9} + \frac{\left(-15\times10^3\right)12}{0.9} + \frac{\left(30\times10^3\right)16}{0.3}\right]$$

$$= 75.9\times10^{-3}\text{in.}$$

$$\boxed{\delta = 75.9\times10^{-3}\text{ in.}}$$

Sample Problem 2.1



The rigid bar *BDE* is supported by two links *AB* and *CD*.

Link *AB* is made of aluminum (*E* = 70 GPa) and has a cross-sectional area of 500 mm^2. Link *CD* is made of steel (*E* = 200 GPa) and has a cross-sectional area of (600 mm^2).

For the 30-kN force shown, determine the deflection a) of *B*, b) of *D*, and c) of *E*.

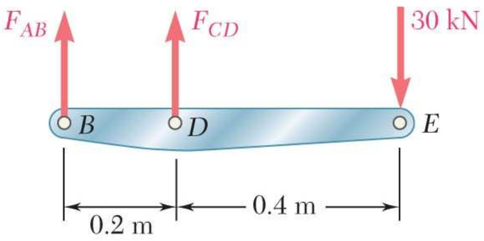
SOLUTION:

- Apply a free-body analysis to the bar *BDE* to find the forces exerted by links *AB* and *DC*.
- Evaluate the deformation of links *AB* and *DC* or the displacements of *B* and *D*.
- Work out the geometry to find the deflection at E given the deflections at B and D.

Sample Problem 2.1

SOLUTION:

Free body: Bar *BDE*



$$+\circlearrowleft \sum M_B = 0$$

$$0 = -(30\,\text{kN} \times 0.6\,\text{m}) + F_{CD} \times 0.2\,\text{m}$$

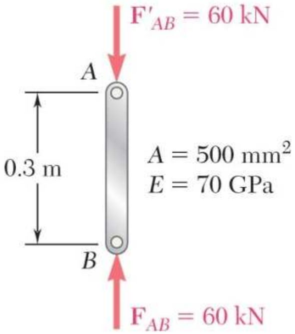
$$F_{CD} = +90\,\text{kN} \quad \textit{tension}$$

$$+\circlearrowleft \sum M_D = 0$$

$$0 = -(30\,\text{kN} \times 0.4\,\text{m}) - F_{AB} \times 0.2\,\text{m}$$

$$F_{AB} = -60\,\text{kN} \quad \textit{compression}$$

Displacement of *B*:



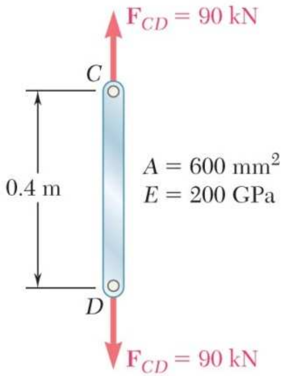
$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60 \times 10^3\,\text{N})(0.3\,\text{m})}{(500 \times 10^{-6}\,\text{m}^2)(70 \times 10^9\,\text{Pa})}$$

$$= -514 \times 10^{-6}\,\text{m}$$

$$\boxed{\delta_B = 0.514\,\text{mm} \uparrow}$$

Displacement of *D*:



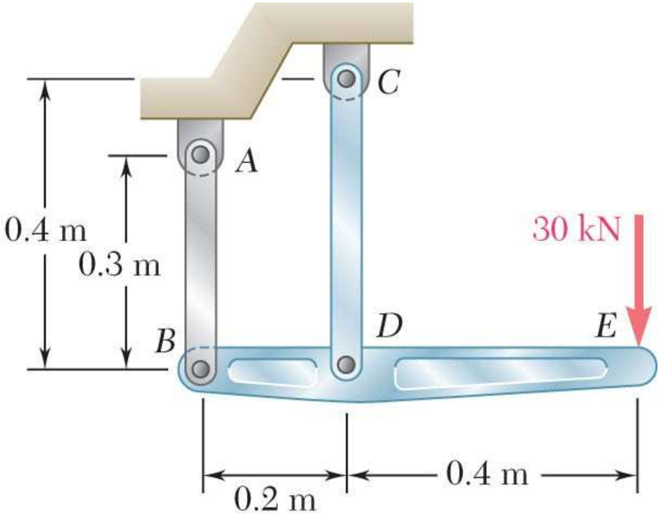
$$\delta_D = \frac{PL}{AE}$$

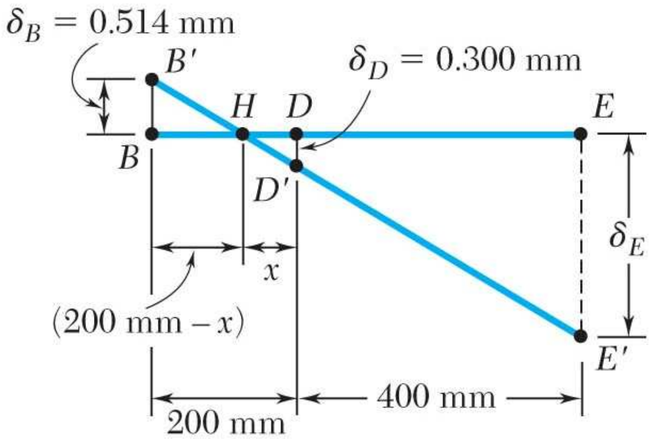
$$= \frac{(90 \times 10^3\,\text{N})(0.4\,\text{m})}{(600 \times 10^{-6}\,\text{m}^2)(200 \times 10^9\,\text{Pa})}$$

$$= 300 \times 10^{-6}\,\text{m}$$

$$\boxed{\delta_D = 0.300\,\text{mm} \downarrow}$$

Sample Problem 2.1





Displacement of D:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

$$\frac{0.514\text{ mm}}{0.300\text{ mm}} = \frac{(200\text{ mm}) - x}{x}$$

$$x = 73.7\text{ mm}$$

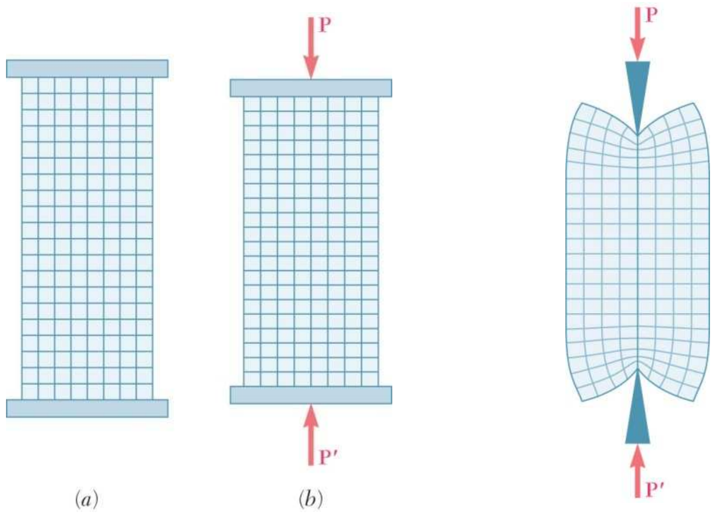
$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

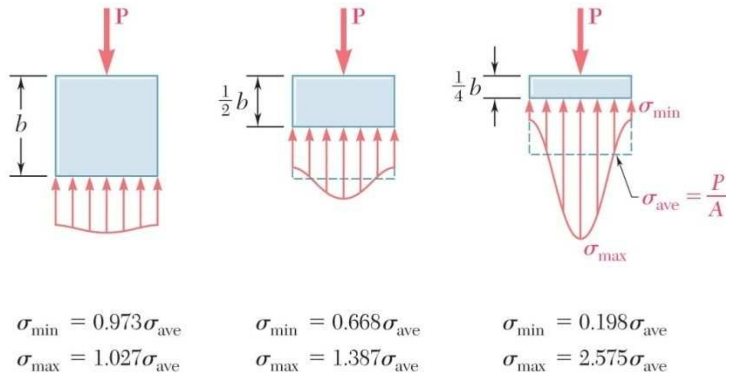
$$\frac{\delta_E}{0.300\text{ mm}} = \frac{(400 + 73.7)\text{mm}}{73.7\text{ mm}}$$

$$\delta_E = 1.928\text{ mm}$$

$$\boxed{\delta_E = 1.928\text{ mm} \downarrow}$$

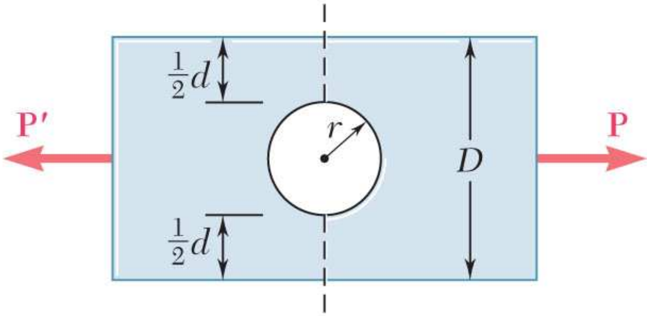
Saint-Venant's Principle

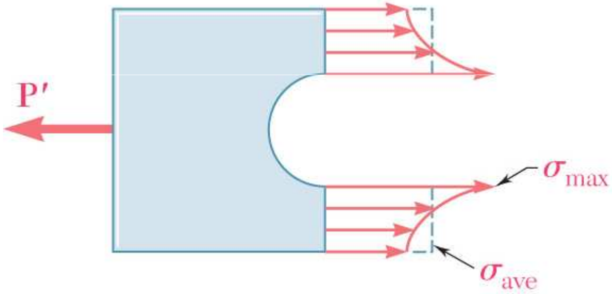


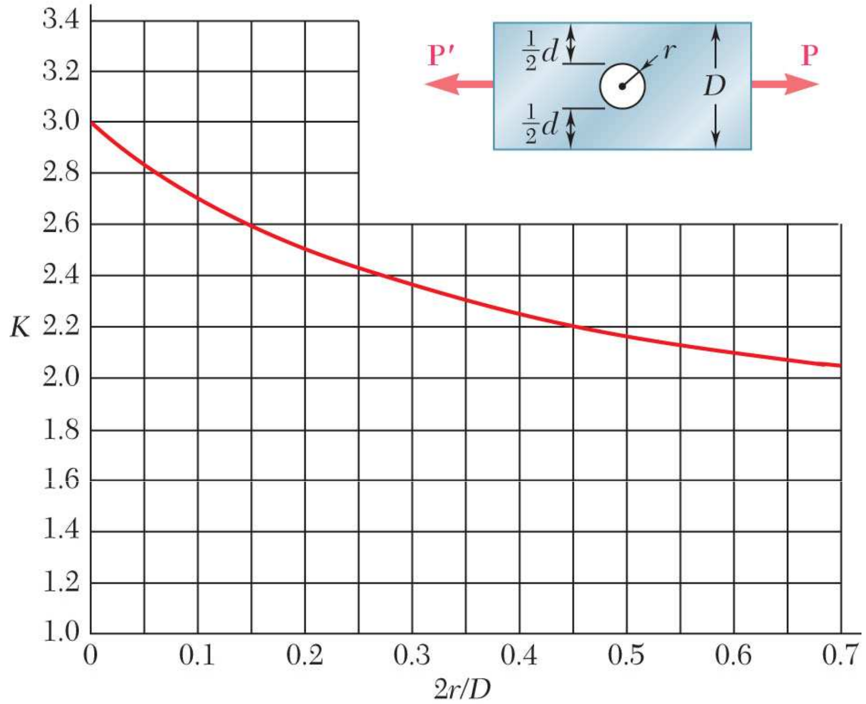


- Loads transmitted through rigid plates result in uniform distribution of stress and strain.
- Concentrated loads result in large stresses in the vicinity of the load application point.
- Stress and strain distributions become uniform at a relatively short distance from the load application points.
- **Saint-Venant's Principle:** Stress distribution may be assumed independent of the mode of load application except in the immediate vicinity of load application points.

Stress Concentration: Hole





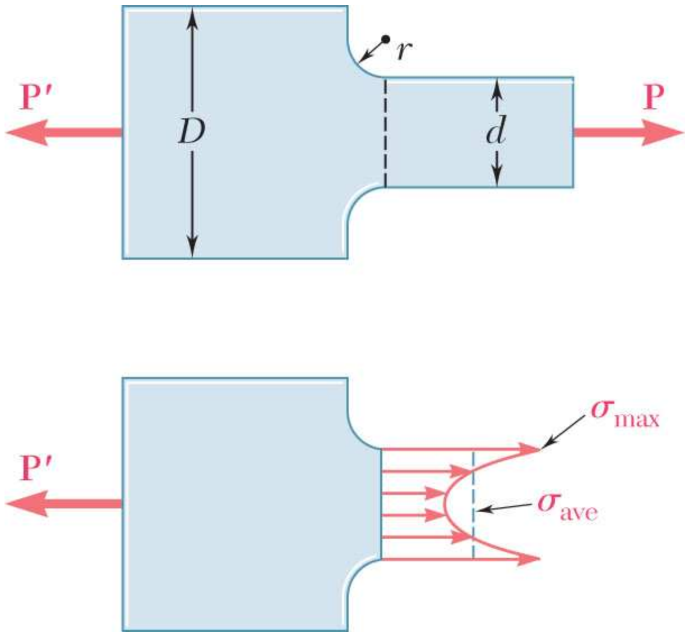


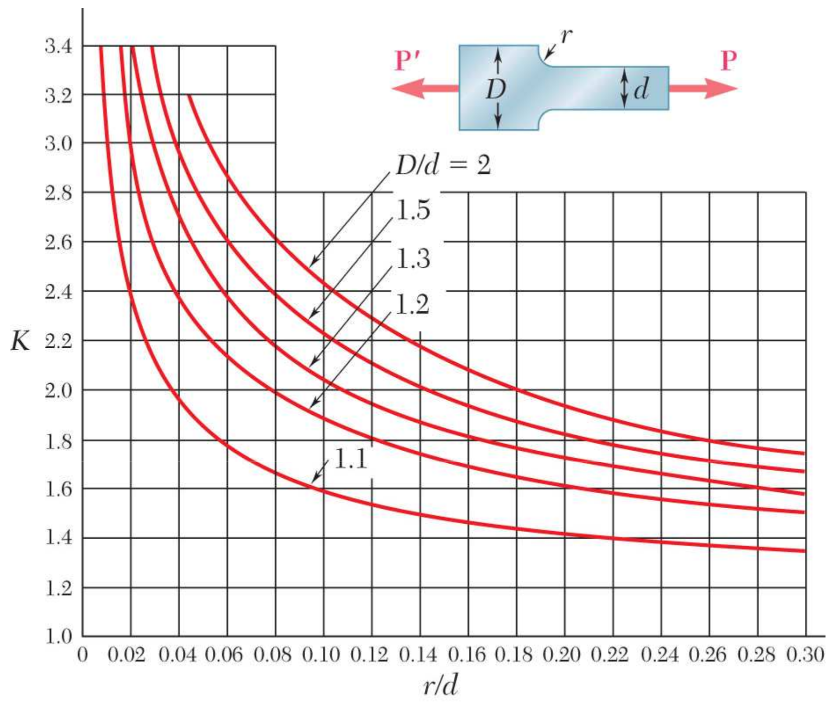
(a) **Flat bars with holes**

Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

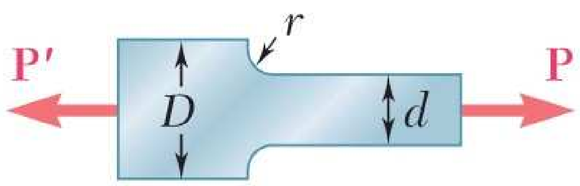
Stress Concentration: Fillet





(b) **Flat bars with fillets**

Example 2.12



Determine the largest axial load *P* that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick, and respectively 40 and 60 mm wide, connected by fillets of radius *r* = 8 mm. Assume an allowable normal stress of 165 MPa.

SOLUTION:

- Determine the geometric ratios and find the stress concentration factor from Fig. 2.64*b*.
- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.
- Apply the definition of normal stress to find the allowable load.