Trusses
Basics & Assumptions

• Trusses are composed of
  ▪ Members (long & slender)
  ▪ Joints (where members connect)
  ▪ Forces are only at Joints
Roof truss
Assumptions

• Pin connections to members at joints.
• **All loads are at joints.**
• All members are **rigid** and **straight**.
  - **Members** are considered **two-force members**
  - Either in **tension** (pulling at joint), OR in **compression** (pushing at joint)
  - Assume tension if unknown.
• Force in member is along the member
Bridge truss

(b)

Figure: 05-002b
Figure: 05-014a

Internal tensile forces

Tension
Example

\[ B \]

\[
\begin{align*}
   &\text{500 N} \\
   \text{2 m} \\
   A \\
   \text{2 m} \\
   C \\
   45^\circ
\end{align*}
\]
Joint B

- Tensions Pulls
- Compression Pushes

\[ F_{BA} \text{(tension)} \]

\[ F_{BC} \text{(compression)} \]

500 N

45°
Section around B

- Tensions Pulls
- Compression Pushes
Joints & Members
- Tensions Pulls
- Compression Pushes
Two approaches to find forces in members

• Method of Joints
  \[ \Sigma F_x = 0 \]
  \[ \Sigma F_y = 0 \]
  OR
• Method of Sections
  \[ \Sigma F_x = 0 \]
  \[ \Sigma F_y = 0 \]
  \[ \Sigma M_z = 0 \]
EXAMPLE 5.3

Determine the force in each member of the truss shown in Fig. 5–10a. Indicate whether the members are in tension or compression.
EXAMPLE 5.3 CONTINUED

SOLUTION

Support Reactions. No joint can be analyzed until the support reactions are determined, because each joint has more than three unknown forces acting on it. A free-body diagram of the entire truss is given in Fig. 5–10b. Applying the equations of equilibrium, we have

\[ \sum F_x = 0; \quad 600 \text{N} - C_x = 0 \quad C_x = 600 \text{N} \]

\[ \sum M_C = 0; \quad -A_y(6 \text{ m}) + 400 \text{N}(3 \text{ m}) + 600 \text{N}(4 \text{ m}) = 0 \quad A_y = 600 \text{N} \]

\[ \sum F_y = 0; \quad 600 \text{N} - 400 \text{N} - C_y = 0 \quad C_y = 200 \text{N} \]

The analysis can now start at either joint A or C. The choice is arbitrary since there are one known and two unknown member forces acting on the pin at each of these joints.

Joint A. (Fig. 5–10c). As shown on the free-body diagram, \( F_{AB} \) is assumed to be compressive and \( F_{AD} \) is tensile. Applying the equations of equilibrium, we have

\[ \sum F_y = 0; \quad 600 \text{N} - \frac{4}{5}F_{AB} = 0 \quad F_{AB} = 750 \text{N} \quad \text{(C) \ Ans.} \]

\[ \sum F_x = 0; \quad F_{AD} - \frac{3}{5}(750 \text{N}) = 0 \quad F_{AD} = 450 \text{N} \quad \text{(T) \ Ans.} \]
**Example 5.3 Continued**

**Joint D.** (Fig. 5–10d). Using the result for \( F_{AD} \) and summing forces in the horizontal direction, Fig. 5–10d, we have

\[
\sum F_x = 0; \quad -450 \text{ N} + \frac{3}{5} F_{DB} + 600 \text{ N} = 0 \quad F_{DB} = -250 \text{ N}
\]

The negative sign indicates that \( F_{DB} \) acts in the opposite sense to that shown in Fig. 5–10d.* Hence,

\[
F_{DB} = 250 \text{ N} \quad (\text{T}) \quad \text{Ans.}
\]

To determine \( F_{DC} \), we can either correct the sense of \( F_{DB} \) on the free-body diagram, and then apply \( \sum F_y = 0 \), or apply this equation and retain the negative sign for \( F_{DB} \), i.e.,

\[
\sum F_y = 0; \quad -F_{DC} - \frac{4}{5}(-250 \text{ N}) = 0 \quad F_{DC} = 200 \text{ N} \quad (\text{C}) \quad \text{Ans.}
\]
**Joint C.** (Fig. 5–10e).

\[ \pm \sum F_x = 0; \quad F_{CB} - 600 \text{ N} = 0 \quad F_{CB} = 600 \text{ N} \quad \text{(C)} \quad \text{Ans.} \]

\[ + \sum F_y = 0; \quad 200 \text{ N} - 200 \text{ N} = 0 \quad \text{(check)} \]

**NOTE:** The analysis is summarized in Fig. 5–10f, which shows the free-body diagram for each joint and member.

*The proper sense could have been determined by inspection, prior to applying \( \sum F_x = 0 \).*
Zero Force Members
\[ \Sigma F_x = 0; \quad F_{AB} = 0 \]
\[ \Sigma F_y = 0; \quad F_{AF} = 0 \]
Zero Force Members
\[ + \downarrow \sum F_y = 0; \quad F_{DC} \sin \theta = 0; \quad F_{DC} = 0 \text{ since } \sin \theta \neq 0 \\
+ \leftarrow \sum F_x = 0; \quad F_{DE} + 0 = 0; \quad F_{DE} = 0 \]
Simplified Truss Design w/o Zero Force Members
Can you spot the Zero Force Members?
Find the Zero force members
EXEMPLARY 5.4

Using the method of joints, determine all the zero-force members of the *Fink roof truss* shown in Fig. 5–13a. Assume all joints are pin connected.

SOLUTION

Look for joint geometries that have three members for which two are collinear. We have

**Joint G.** (Fig. 5–13b).

\[ + \uparrow \sum F_y = 0; \quad F_{GC} = 0 \]

*Ans.*
EXAMPLE 5.4 CONTINUED

Realize that we could not conclude that GC is a zero-force member by considering joint C, where there are five unknowns. The fact that GC is a zero-force member means that the 5-kN load at C must be supported by members CB, CH, CF, and CD.

**Joint D.** (Fig. 5–13c).

\[ + \sqrt{\sum F_x} = 0; \quad F_{DF} = 0 \quad \text{Ans.} \]

**Joint F.** (Fig. 5–13d).

\[ + \uparrow \sum F_y = 0; \quad F_{FC} \cos \theta = 0 \quad \text{Since} \, \theta \neq 90^\circ, \quad F_{FC} = 0 \quad \text{Ans.} \]

**NOTE:** If joint B is analyzed, Fig. 5–13e,

\[ + \downarrow \sum F_x = 0; \quad 2 \, \text{kN} - F_{BH} = 0 \quad F_{BH} = 2 \, \text{kN} \quad \text{(C)} \]

Also, \( F_{HC} \) must satisfy \( \sum F_y = 0 \), Fig. 5–13f, and therefore HC is not a zero-force member.
Method of Sections

A 2 m 2 m 2 m

1000 N

B a C

G a F

D 2 m

(a)
Internal forces

Internal tensile forces

Tension

Internal compressive forces

Compression
Method of Sections: 3 unknowns Max
EXAMPLE 5.5

Determine the force in members GE, GC, and BC of the truss shown in Fig. 5–16a. Indicate whether the members are in tension or compression.

SOLUTION

Section aa in Fig. 5–16a has been chosen since it cuts through the three members whose forces are to be determined. In order to use the method of sections, however, it is first necessary to determine the external reactions at A or D. Why? A free-body diagram of the entire truss is shown in Fig. 5–16b. Applying the equations of equilibrium, we have

\[ \sum F_x = 0; \quad 400 \text{ N} - A_x = 0 \quad A_x = 400 \text{ N} \]

\[ \downarrow \sum M_A = 0; \quad -1200 \text{ N}(8 \text{ m}) - 400 \text{ N}(3 \text{ m}) + D_y(12 \text{ m}) = 0 \]

\[ D_y = 900 \text{ N} \]

\[ \uparrow \sum F_y = 0; \quad A_y - 1200 \text{ N} + 900 \text{ N} = 0 \]

\[ A_y = 300 \text{ N} \]
**FREE-BODY DIAGRAM.** For the analysis the free-body diagram of the left portion of the sectioned truss will be used, since it involves the least number of forces, Fig. 5–16c.

**EQUATIONS OF EQUILIBRIUM.** Summing moments about point G eliminates \( F_{GE} \) and \( F_{GC} \) and yields a direct solution for \( F_{BC} \).

\[
\sum M_G = 0; \quad -300 \text{ N}(4 \text{ m}) - 400 \text{ N}(3 \text{ m}) + F_{BC}(3 \text{ m}) = 0
\]

\[
F_{BC} = 800 \text{ N} \quad \text{(T)} \quad \text{Ans.}
\]

In the same manner, by summing moments about point C we obtain a direct solution for \( F_{GE} \).

\[
\sum M_C = 0; \quad -300 \text{ N}(8 \text{ m}) + F_{GE}(3 \text{ m}) = 0
\]

\[
F_{GE} = 800 \text{ N} \quad \text{(C)} \quad \text{Ans.}
\]

Since \( F_{BC} \) and \( F_{GE} \) have no vertical components, summing forces in the y direction directly yields \( F_{GC} \), i.e.,

\[
\sum F_y = 0; \quad 300 \text{ N} - \frac{3}{5} F_{GC} = 0
\]

\[
F_{GC} = 500 \text{ N} \quad \text{(T)} \quad \text{Ans.}
\]

**NOTE:** Here it is possible to tell, by inspection, the proper direction for each unknown member force. For example, \( \sum M_C = 0 \) requires \( F_{GE} \) to be compressive because it must balance the moment of the 300-N force about C.
Machines/Frames

100 N

250 mm

50 mm

45°
EXAMPLE 5.7

For the frame shown in Fig. 5–18a, draw the free-body diagram of (a) each member, (b) the pin at B, and (c) the two members connected together.

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SOLUTION

Part (a). By inspection, members BA and BC are not two-force members. Instead, as shown on the free-body diagrams, Fig. 5–18b, BC is subjected to a force from the pins at B and C and the external force P. Likewise, AB is subjected to a force from the pins at A and B and the external couple moment M. The pin forces are represented by their \( x \) and \( y \) components.

Part (b). The pin at B is subjected to only two forces, i.e., the force of member BC and the force of member AB. For equilibrium these forces or their respective components must be equal but opposite, Fig. 5–18c. Realize that Newton's third law is applied between the pin and its connected members, i.e., the effect of the pin on the two members, Fig. 5–18b, and the equal but opposite effect of the two members on the pin, Fig. 5–18c.

Part (c). The free-body diagram of both members connected together, yet removed from the supporting pins at A and C, is shown in Fig. 5–18d. The force components \( B_x \) and \( B_y \) are not shown on this diagram since they are internal forces (Fig. 5–18b) and therefore cancel out. Also, to be consistent when later applying the equilibrium equations, the unknown force components at A and C must act in the same sense as those shown in Fig. 5–18b.
For the frame shown in Fig. 5–19a, draw the free-body diagrams of (a) the entire frame including the pulleys and cords, (b) the frame without the pulleys and cords, and (c) each of the pulleys.

(a)
EXAMPLE 5.8 CONTINUED

SOLUTION

Part (a). When the entire frame including the pulleys and cords is considered, the interactions at the points where the pulleys and cords are connected to the frame become pairs of *internal* forces which cancel each other and therefore are not shown on the free-body diagram, Fig. 5–19b.

Part (b). When the cords and pulleys are removed, their effect *on the frame* must be shown, Fig. 5–19c.

Part (c). The force components $B_x$, $B_y$, $C_x$, $C_y$ of the pins on the pulleys, Fig. 5–19d, are equal but opposite to the force components exerted by the pins on the frame, Fig. 5–19c. Why?
EXAMPLE 5.9

Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame in Fig. 5–20a.

SOLUTION I

Free-Body Diagrams. By inspection it can be seen that AB is a two-force member. The free-body diagrams are shown in Fig. 5–20b.

Equations of Equilibrium. The three unknowns can be determined by applying the three equations of equilibrium to member CB.

\[
\begin{align*}
\downarrow \sum M_C &= 0; \quad 2000 \text{ N}(2 \text{ m}) - (F_{AB} \sin 60^\circ)(4 \text{ m}) = 0 \quad F_{AB} = 1154.7 \text{ N} \\
\uparrow \sum F_x &= 0; \quad 1154.7 \cos 60^\circ \text{ N} - C_x = 0 \quad C_x = 577 \text{ N} \quad \text{Ans.} \\
+ \sum F_y &= 0; \quad 1154.7 \sin 60^\circ \text{ N} - 2000 \text{ N} + C_y = 0 \quad C_y = 1000 \text{ N} \quad \text{Ans.}
\end{align*}
\]

SOLUTION II

Free-Body Diagrams. If one does not recognize that AB is a two-force member, then more work is involved in solving this problem. The free-body diagrams are shown in Fig. 5–20c.
**Equations of Equilibrium.** The six unknowns are determined by applying the three equations of equilibrium to each member.

**Member AB**
\[ \sum M_A = 0; \quad B_x(3 \sin 60^\circ \text{ m}) - B_y(3 \cos 60^\circ \text{ m}) = 0 \]  
\[ \sum F_x = 0; \quad A_x - B_x = 0 \]  
\[ \sum F_y = 0; \quad A_y - B_y = 0 \]  

**Member BC**
\[ \sum M_C = 0; \quad 2000 \text{ N}(2 \text{ m}) - B_y(4 \text{ m}) = 0 \]  
\[ \sum F_x = 0; \quad B_x - C_x = 0 \]  
\[ \sum F_y = 0; \quad B_y - 2000 \text{ N} + C_y = 0 \]

The results for \( C_x \) and \( C_y \) can be determined by solving these equations in the following sequence: 4, 1, 5, then 6. The results are

\[ B_y = 1000 \text{ N} \]  
\[ B_x = 577 \text{ N} \]  
\[ C_x = 577 \text{ N} \]  
\[ C_y = 1000 \text{ N} \]  

By comparison, Solution I is simpler since the requirement that \( F_{AB} \) in Fig. 5–20b be equal, opposite, and collinear at the ends of member AB automatically satisfies Eqs. 1, 2, and 3 above and therefore eliminates the need to write these equations. As a result, save yourself some time and effort by always identifying the two-force members before starting the analysis!
EXAMPLE 5.12

The smooth disk shown in Fig. 5–23a is pinned at D and has a weight of 20 lb. Neglecting the weights of the other members, determine the horizontal and vertical components of reaction at pins B and D.

SOLUTION

Free-Body Diagrams. The free-body diagrams of the entire frame and each of its members are shown in Fig. 5–23b.

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EXAMPLE 5.12 CONTINUED

Equations of Equilibrium. The eight unknowns can of course be obtained by applying the eight equilibrium equations to each member—three to member $AB$, three to member $BCD$, and two to the disk. (Moment equilibrium is automatically satisfied for the disk.) If this is done, however, all the results can be obtained only from a simultaneous solution of some of the equations. (Try it and find out.) To avoid this situation, it is best first to determine the three support reactions on the entire frame; then, using these results, the remaining five equilibrium equations can be applied to two other parts in order to solve successively for the other unknowns.

**Entire Frame**

\[ \Sigma M_A = 0; \quad -20 \text{ lb (3 ft)} + C_x(3.5 \text{ ft}) = 0 \quad C_x = 17.1 \text{ lb} \]

\[ \Sigma F_x = 0; \quad A_x - 17.1 \text{ lb} = 0 \quad A_x = 17.1 \text{ lb} \]

\[ \Sigma F_y = 0; \quad A_y - 20 \text{ lb} = 0 \quad A_y = 20 \text{ lb} \]

**Member AB**

\[ \Sigma F_x = 0; \quad 17.1 \text{ lb} - B_x = 0 \quad B_x = 17.1 \text{ lb} \quad \text{Ans.} \]

\[ \Sigma M_B = 0; \quad -20 \text{ lb (6 ft)} + N_D(3 \text{ ft}) = 0 \quad N_D = 40 \text{ lb} \]

\[ \Sigma F_y = 0; \quad 20 \text{ lb} - 40 \text{ lb} + B_y = 0 \quad B_y = 20 \text{ lb} \quad \text{Ans.} \]

**Disk**

\[ \Sigma F_x = 0; \quad D_x = 0 \quad \text{Ans.} \]

\[ \Sigma F_y = 0; \quad 40 \text{ lb} - 20 \text{ lb} - D_y = 0 \quad D_y = 20 \text{ lb} \quad \text{Ans.} \]