Today’s Objectives:
Students will be able to:
1. Resolve the acceleration of a point on a body into components of translation and rotation.
2. Determine the acceleration of a point on a body by using a relative acceleration analysis.

In-Class Activities:
• Check Homework
• Reading Quiz
• Applications
• Translation and Rotation Components of Acceleration
• Relative Acceleration Analysis
• Roll-Without-Slip Motion
• Concept Quiz
• Group Problem Solving
• Attention Quiz
1. If two bodies contact one another without slipping, and the points in contact move along different paths, the tangential components of acceleration will be ______ and the normal components of acceleration will be _________.
   A) the same, the same       B) the same, different
   C) different, the same      D) different, different

2. When considering a point on a rigid body in general plane motion,
   A) It’s total acceleration consists of both absolute acceleration and relative acceleration components.
   B) It’s total acceleration consists of only absolute acceleration components.
   C) It’s relative acceleration component is always normal to the path.
   D) None of the above.
In the mechanism for a window, link AC rotates about a fixed axis through C, and AB undergoes general plane motion. Since point A moves along a curved path, it has two components of acceleration while point B, sliding in a straight track, has only one.

The components of acceleration of these points can be inferred since their motions are known.

How can we determine the accelerations of the links in the mechanism?
In an automotive engine, the forces delivered to the crankshaft, and the angular acceleration of the crankshaft, depend on the speed and acceleration of the piston.

How can we relate the accelerations of the piston, connection rod, and crankshaft to each other?
The equation relating the accelerations of two points on the body is determined by differentiating the velocity equation with respect to time.

\[
\frac{dv_B}{dt} = \frac{dv_A}{dt} + \frac{dv_{B/A}}{dt}
\]

These are absolute accelerations of points A and B. They are measured from a set of fixed x,y axes.

This term is the acceleration of B with respect to A and includes both tangential and normal components.

The result is \( a_B = a_A + (a_{B/A})_t + (a_{B/A})_n \)
The relative tangential acceleration component \((\mathbf{a}_{B/A})_t\) is \((\mathbf{\alpha} \times \mathbf{r}_{B/A})\) and perpendicular to \(\mathbf{r}_{B/A}\).

The relative normal acceleration component \((\mathbf{a}_{B/A})_n\) is \((-\omega^2 \mathbf{r}_{B/A})\) and the direction is always from B towards A.
Since the relative acceleration components can be expressed as \((a_{B/A})_t = \alpha \times r_{B/A}\) and \((a_{B/A})_n = - \omega^2 r_{B/A}\), the relative acceleration equation becomes

\[
a_{B} = a_{A} + \alpha \times r_{B/A} - \omega^2 r_{B/A}
\]

Note that the last term in the relative acceleration equation is not a cross product. It is the product of a scalar (square of the magnitude of angular velocity, \(\omega^2\)) and the relative position vector, \(r_{B/A}\).
In applying the relative acceleration equation, the two points used in the analysis (A and B) should generally be selected as points which have a known motion, such as pin connections with other bodies.

In this mechanism, point B is known to travel along a circular path, so \( \mathbf{a}_B \) can be expressed in terms of its normal and tangential components. Note that point B on link BC will have the same acceleration as point B on link AB.

Point C, connecting link BC and the piston, moves along a straight-line path. Hence, \( \mathbf{a}_C \) is directed horizontally.
1. Establish a fixed coordinate system.

2. Draw the kinematic diagram of the body.

3. Indicate on it $\mathbf{a}_A$, $\mathbf{a}_B$, $\omega$, $\alpha$, and $\mathbf{r}_{B/A}$. If the points A and B move along curved paths, then their accelerations should be indicated in terms of their tangential and normal components, i.e., $\mathbf{a}_A = (\mathbf{a}_A)_t + (\mathbf{a}_A)_n$ and $\mathbf{a}_B = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n$.

4. Apply the relative acceleration equation:

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

5. If the solution yields a negative answer for an unknown magnitude, this indicates that the sense of direction of the vector is opposite to that shown on the diagram.
Example 1

Given: Point A on rod AB has an acceleration of 5 m/s² and a velocity of 6 m/s at the instant shown.

Find: The angular acceleration of the rod and the acceleration at B at this instant.

Plan: Follow the problem solving procedure!

Solution: First, we need to find the angular velocity of the rod at this instant. Locating the instant center (IC) for rod AB, we can determine \( \omega \):

\[
\omega = \frac{v_A}{r_{A/IC}} = \frac{6}{3} = 2 \text{ rad/s}
\]
Since points A and B both move along straight-line paths,

\[ a_A = -5 \, j \, \text{m/s}^2 \]

\[ a_B = a_B \, i \, \text{m/s}^2 \]

Applying the relative acceleration equation

\[ \mathbf{a}_B = \mathbf{a}_A + \mathbf{a} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A} \]

\[ a_B i = -5 j + \alpha k \times (3 i - 4 j) - 2^2 (3 i - 4 j) \]

\[ a_B i = -5 j + 4 \alpha i + 3 \alpha j + (-12 i + 16 j) \]

\[ = (4 \alpha - 12) i + (3 \alpha + 11) j \]
So with \( a_B \mathbf{i} = (4 \alpha - 12) \mathbf{i} + (3 \alpha + 11) \mathbf{j} \), we can solve for \( a_B \) and \( \alpha \).

By comparing the \( i, j \) components;

\[
\begin{align*}
a_B &= 4 \alpha - 12 \\
0 &= 3\alpha + 11
\end{align*}
\]

Solving:

\[
\begin{align*}
a_B &= -26.7 \text{ m/s}^2 \\
\alpha &= -3.67 \text{ rad/s}^2
\end{align*}
\]
Consider two bodies in contact with one another without slipping, where the points in contact move along different paths.

In this case, the **tangential components** of acceleration will be the same, i.e.,

\[(a_A)_t = (a_{A'})_t\] (which implies \(\alpha_B \times r_B = \alpha_C \times r_C\)).

The **normal components** of acceleration will not be the same.

\[(a_A)_n \neq (a_{A'})_n\] so \(a_A \neq a_{A'}\).
Another common type of problem encountered in dynamics involves rolling motion without slip; e.g., a ball, cylinder, or disk rolling without slipping. This situation can be analyzed using relative velocity and acceleration equations.

As the cylinder rolls, point G (center) moves along a straight line. If $\omega$ and $\alpha$ are known, the relative velocity and acceleration equations can be applied to A, at the instant A is in contact with the ground. The point A is the instantaneous center of zero velocity, however it is not a point of zero acceleration.
ROLLING MOTION (continued)

- **Velocity:** Since no slip occurs, $v_A = 0$ when A is in contact with ground. From the kinematic diagram:
  $$v_G = v_A + \omega \times r_{G/A}$$
  $$v_G \mathbf{i} = 0 + (-\omega \mathbf{k}) \times (r \mathbf{j})$$
  $$v_G = \omega r \quad \text{or} \quad v_G = \omega r \mathbf{i}$$

- **Acceleration:** Since G moves along a straight-line path, $a_G$ is horizontal. Just before A touches ground, its velocity is directed downward, and just after contact, its velocity is directed upward. Thus, point A accelerates upward as it leaves the ground.
  $$a_G = a_A + \alpha \times r_{G/A} - \omega^2 r_{G/A} \quad \Rightarrow \quad a_G \mathbf{i} = a_A \mathbf{j} + (-\alpha \mathbf{k}) \times (r \mathbf{j}) - \omega^2 (r \mathbf{j})$$

Evaluating and equating $i$ and $j$ components:
  $$a_G = \alpha r \quad \text{and} \quad a_A = \omega^2 r \quad \text{or} \quad a_G = \alpha r \mathbf{i} \quad \text{and} \quad a_A = \omega^2 r \mathbf{j}$$
**Given:** The gear with a center at O rolls on the fixed rack.

**Find:** The acceleration of point A at this instant.

**Plan:**
Follow the solution procedure!

**Solution:** Since the gear rolls on the fixed rack without slip, $a_O$ is directed to the right with a magnitude of

$$a_O = \alpha r = (6 \text{ rad/s}^2) (0.3 \text{ m}) = 1.8 \text{ m/s}^2.$$
So now with $a_O = 1.8 \text{ m/s}^2$, we can apply the relative acceleration equation between points O and A.

$$a_A = a_O + \alpha \times r_{A/O} - \omega^2 r_{A/O}$$

$$a_A = 1.8 \hat{i} + (-6 \hat{k}) \times (0.3 \hat{j}) - 12^2 (0.3 \hat{j})$$

$$a_A = (3.6 \hat{i} - 43.2 \hat{j}) \text{ m/s}^2$$
CONCEPT QUIZ

1. If a ball rolls without slipping, select the tangential and normal components of the relative acceleration of point A with respect to G.

   A) \( \alpha r \, i + \omega^2 r \, j \)          B) \(- \alpha r \, i + \omega^2 r \, j \)
   C) \( \omega^2 r \, i - \alpha r \, j \)          D) Zero.

2. What are the tangential and normal components of the relative acceleration of point B with respect to G.

   A) \(- \omega^2 r \, i - \alpha r \, j \)          B) \(- \alpha r \, i + \omega^2 r \, j \)
   C) \( \omega^2 r \, i - \alpha r \, j \)          D) Zero.
GROUP PROBLEM SOLVING

Given: Member AB is rotating with \( \omega_{AB}=4 \text{ rad/s}, \alpha_{AB}=5 \text{ rad/s}^2 \) at this instant.

Find: The velocity and acceleration of the slider block C.

Plan: Follow the solution procedure!

Note that Point B is rotating about A. So what components of acceleration will it be experiencing?
Solution:
Since Point B is rotating, its velocity and acceleration will be:

\[ \mathbf{v}_B = (\omega_{AB}) \mathbf{r}_{B/A} = (4)(2) = 8 \text{ m/s} \]
\[ \mathbf{a}_{Bn} = (\omega_{AB})^2 \mathbf{r}_{B/A} = (4)^2(2) = 32 \text{ m/s}^2 \]
\[ \mathbf{a}_{Bt} = (\alpha_{AB}) \mathbf{r}_{B/A} = (-5)(2) = -10 \text{ m/s}^2 \]

Thus:
\[ \mathbf{v}_B = (-8 \mathbf{i}) \text{ m/s} \]
\[ \mathbf{a}_B = (10 \mathbf{i} - 32 \mathbf{j}) \text{ m/s}^2 \]
Now apply the relative velocity equation between points B and C to find the angular velocity of link BC.

\[ v_C = v_B + \omega_{BC} \times r_{C/B} \]

\[ (-0.8 \, \hat{i} - 0.6 \, \hat{j}) \, v_C = (-8 \, \hat{i}) + \omega_{BC} \, \hat{k} \times (-0.5 \, \hat{i} - 2 \, \hat{j}) \]

\[ (-0.8 \, \hat{i} - 0.6 \, \hat{j}) \, v_C = (-8 + 2 \, \omega_{BC}) \, \hat{i} - 0.5 \, \omega_{BC} \, \hat{j} \]

By comparing the \( i, j \) components:

\[-0.8 \, v_C = -8 + 2 \, \omega_{BC} \]
\[-0.6 \, v_C = -0.5 \, \omega_{BC} \]

Solving:

\[ \omega_{BC} = 3 \, \text{rad/s} \]
\[ v_C = 2.50 \, \text{m/s} \]
Now, apply the relative acceleration equation between points B and C.

\[ a_C = a_B + \alpha_{BC} \times r_{C/B} - \omega_{BC}^2 r_{C/B} \]

\([-0.8 \, i - 0.6 \, j]\) \[a_C = (10 \, i - 32 \, j) + \alpha_{BC} \, k \times (-0.5 \, i - 2 \, j) \]

\[-(3)^2 (-0.5 \, i - 2 \, j)\]

\([-0.8 \, i - 0.6 \, j]\) \[a_C = (10 \, i - 32 \, j) + (2\alpha_{BC} \, i - 0.5\alpha_{BC} \, j) \]

\[+ (4.5 \, i + 18 \, j)\]

By comparing the \(i, j\) components;

- \(-0.8 \, a_C = 14.5 + 2 \, \alpha_{BC}\)
- \(-0.6 \, a_C = -14 - 0.5 \, \alpha_{BC}\)
GROUP PROBLEM SOLVING (continued)

Solving these two $i, j$ component equations for $a_A$ and $\alpha_{AB}$ yields:

- $0.8 \ a_C = 14.5 + 2 \ \alpha_{BC}$
- $0.6 \ a_C = -14 - 0.5 \ \alpha_{BC}$

$\alpha_{BC} = -12.4 \ \text{rad/s}^2 = 12.4 \ \text{rad/s}^2$

$a_C = 13.0 \ \text{m/s}^2$
1. Two bodies contact one another without slipping. If the tangential component of the acceleration of point A on gear B is 100 ft/sec², determine the tangential component of the acceleration of point A’ on gear C.

A) 50 ft/sec²  B) 100 ft/sec²  
C) 200 ft/sec²  D) None of above.

2. If the tangential component of the acceleration of point A on gear B is 100 ft/sec², determine the angular acceleration of gear B.

A) 50 rad/sec²  B) 100 rad/sec²  
C) 200 rad/sec²  D) None of above.