1. Convert the following system into state space formulation.

\[ g(s) = \frac{s + a}{(s + 2)(s + b)} \]

\[ A = \begin{bmatrix} 0 & 1 \\ -2b & -2 - b \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} a & 1 \end{bmatrix}; D = [0] \]

2. Determine the characteristic equation and the find the roots of it (find the poles of g(s)).

\[ S^2 + (2+b)s + 2b = 0 \]

S = -2, -b

3. Determine by hand the eigenvalues of the A matrix of the state space formulation.

\[ \lambda = -2, -b \]

4. Using Matlab (not by hand) repeat problems 1 through 3 for a=1 and b=3.

A = -5  -6
   1   0

B = 1
   0

C = 1  1

D = 0

5. Find new values of [A,B,C,D] for the controllable canonical form.

\[ \texttt{csys=canon(sys,'companion')} \]

a = 0  -5
    1  -6

b = 1
  0

c = 1  -4

d = 0

6. The linearized, dynamics of a real hovering helicopter are given below:
The model has 4 state variables:
\[ \theta(t): \text{Fuselage pitch angle (rad)} \]
\[ q(t): \text{Pitch rate (rad/sec)} \]
\[ u(t): \text{Horizontal velocity of CG (m/s)} \]
\[ x(t): \text{Horizontal distance of CG from desired hover point (m)} \]

The control variable is: \[ \delta(t): \text{Tilt angle of the rotor thrust (rad)} \]

The linearized equations of motion are:
\[
\begin{align*}
\dot{\theta}(t) &= q(t) \\
\dot{q}(t) &= -0.415q(t) - 0.011u(t) + 6.27\delta(t) \\
\dot{u}(t) &= 9.8\theta(t) - 1.43q(t) - 0.01998u(t) + 9.8\delta(t) \\
\dot{x}(t) &= u(t)
\end{align*}
\]

a. Write the state space representation of the system dynamics, where
\[ X^T = [\theta \quad q \quad u \quad x]^T. \]

\[
\dot{X} = 
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -0.415 & -0.011 & 0 \\
9.8 & -1.43 & -0.01998 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix} \begin{bmatrix}
\theta \\
q \\
u \\
x \\
\end{bmatrix} + 
\begin{bmatrix}
0 \\
6.27 \\
9.8 \\
0 \\
\end{bmatrix} \delta
\]

\[ y = [0 \quad 0 \quad 0 \quad 1]X + [0]\delta \]

b. Determine the eigenvalues of the system. What can you infer about system stability from the eigenvalues?

c. The eigenvalues of A in rad/s are:
\[ \begin{align*}
0 \\
-0.6794 \\
0.1222 + 0.3791i \\
0.1222 - 0.3791i
\end{align*} \]

The open-loop system is unstable, as there are poles with positive real values.
c. Use Matlab to determine if the system is controllable and observable.

\[
\text{my_ctrl_rank} = 4; \quad \text{my_obsrv_rank} = 4;
\]
The ranks of the controllability matrix and observability matrix are full, so the system is a candidate for full state feedback.

d. Determine the transfer function between the thrust vector tilt angle, \( \delta(t) \), and the hover position, \( x(t) \). That is \( g(s) = \frac{x(s)}{\delta(s)} \).

\[
g(s) = \frac{-2.22e-016 s^3 + 9.8 s^2 - 4.899 s + 61.45}{s^4 + 0.435 s^3 - 0.007438 s^2 + 0.1078 s}
\]

e. What are the poles of \( g(s) \)? How do they compare to the results of part b?

Same as b.