MAE 491 & MAE 591
Mechatronics 2004

Assignment 7, State Space Representation and Eigenvalues
Due Tuesday, March 23, 2004

1. Convert the following system into state space formulation.

\[ g(s) = \frac{s + a}{(s + 2)(s + b)} \]

2. Determine the characteristic equation and find the roots of it (find the poles of \( g(s) \)).
3. Determine by hand the eigenvalues of the \( A \) matrix of the state space formulation.
4. Using Matlab (not by hand) repeat problems 1 through 3 for \( a=1 \) and \( b=3 \).
5. Find new values of \([A,B,C,D]\) for the controllable canonical form.
6. The linearized, dynamics of a real hovering helicopter are given below:

The model has 4 state variables:
- \( \theta(t) \): Fuselage pitch angle (rad)
- \( q(t) \): Pitch rate (rad/sec)
- \( u(t) \): Horizontal velocity of CG (m/s)
- \( x(t) \): Horizontal distance of CG from desired hover point (m)

The control variable is: \( \delta(t) \): Tilt angle of the rotor thrust (rad)

The linearized equations of motion are:

\[
\begin{align*}
\dot{\theta}(t) &= q(t) \\
\dot{q}(t) &= -0.415q(t) - 0.011u(t) + 6.27\delta(t) \\
\dot{u}(t) &= 9.80(t) - 1.43q(t) - 0.01998u(t) + 9.8\delta(t) \\
\dot{x}(t) &= u(t)
\end{align*}
\]

a. Write the state space representation of the system dynamics, where \( X^T = [\theta \quad q \quad u \quad x]^T \).
b. Determine the eigenvalues of the system. What can you infer about system stability from the eigenvalues?
c. Use Matlab to determine if the system is controllable and observable.
d. Determine the transfer function between the thrust vector tilt angle, \( \delta(t) \), and the hover position, \( x(t) \). That is \( g(s) = x(s) / \delta(s) \).
e. What are the poles of \( g(s) \)? How do they compare to the results of part b?