Threaded Fasteners 2
Bolted Joint Stiffnesses

- During bolt preload
  - bolt is stretched
  - members in grip are compressed
- When external load $P$ is applied
  - Bolt stretches further
  - Members in grip uncompress some
- Joint can be modeled as a soft bolt spring in parallel with a stiff member spring

Fig. 8–13
Bolt Stiffness

- Axially loaded rod, partly threaded and partly unthreaded
- Consider each portion as a spring
- Combine as two springs in series

\[ \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \quad \text{or} \quad k = \frac{k_1k_2}{k_1 + k_2} \]  
\[ k_t = \frac{A_t E}{l_t} \quad k_d = \frac{A_d E}{l_d} \quad \text{unthreaded} \]  
\[ k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} \]  

(8–15)  
(8–16)  
(8–17)
Effective Grip Length for Tapped Holes

- For screw in tapped hole, effective grip length is

\[
  l = \begin{cases} 
    h + t_2/2, & t_2 < d \\ 
    h + d/2, & t_2 \geq d 
  \end{cases}
\]
Procedure to Find Bolt Stiffness

Given fastener diameter $d$ and pitch $p$ in mm or number of threads per inch

Washer thickness: $t$ from Table A–32 or A–33

Nut thickness [Fig. (a) only]: $H$ from Table A–31

Grip length:
   For Fig. (a): $l = \text{thickness of all material squeezed between face of bolt and face of nut}$
   Nut

For Fig. (b):$l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$

Tapped hole

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**Diagram:**
- $l_d$
- $l_i$
- $H$
- $t$
- $h$
- $t_1$
- $t_2$
- $d$
- $L_T$
- $l$
- $L$
- $l_d$
- $L_T$
Procedure to Find Bolt Stiffness

Fastener length (round up using Table A–17*):

For Fig. (a): \( L > l + H \)
For Fig. (b): \( L > h + 1.5d \)

Threaded length \( L_T \):

Inch series:
\[
L_T = \begin{cases} 
2d + \frac{1}{4} \text{ in}, & L \leq 6 \text{ in} \\
2d + \frac{1}{2} \text{ in}, & L > 6 \text{ in}
\end{cases}
\]

Metric series:
\[
L_T = \begin{cases} 
2d + 6 \text{ mm}, & L \leq 125 \text{ mm}, \ d \leq 48 \text{ mm} \\
2d + 12 \text{ mm}, & 125 < L \leq 200 \text{ mm} \\
2d + 25 \text{ mm}, & L > 200 \text{ mm}
\end{cases}
\]
Procedure to Find Bolt Stiffness

Length of unthreaded portion in grip: \[ l_d = L - L_T \]
Length of threaded portion in grip: \[ l_t = l - l_d \]
Area of unthreaded portion: \[ A_d = \pi d^2 / 4 \]
Area of threaded portion: \[ A_t \text{ from Table 8–1 or 8–2} \]
Fastener stiffness: \[ k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} \]
Member Stiffness

- Stress distribution spreads from face of bolt head and nut
- Model as a cone with top cut off
- Called a frustum
**Member Stiffness**

- Model compressed members as if they are frusta spreading from the bolt head and nut to the midpoint of the grip.
- Each frustum has a half-apex angle of $\alpha$.
- Find stiffness for frustum in compression.

---

*Fig. 8–15*
Member Stiffness

\[ d\delta = \frac{P \, dx}{EA} \]

\[ A = \pi \left( r_o^2 - r_i^2 \right) = \pi \left[ \left( x \tan \alpha + \frac{D}{2} \right)^2 - \left( \frac{d}{2} \right)^2 \right] \]

\[ = \pi \left( x \tan \alpha + \frac{D + d}{2} \right) \left( x \tan \alpha + \frac{D - d}{2} \right) \]

\[ \delta = \frac{P}{\pi E} \int_0^t \frac{dx}{[x \tan \alpha + (D + d)/2][x \tan \alpha + (D - d)/2]} \]

\[ \delta = \frac{P}{\pi Ed \tan \alpha} \ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)} \]

\[ k = \frac{P}{\delta} = \frac{\pi Ed \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}} \]

(8-19)
Member Stiffness

- With typical value of $\alpha = 30^\circ$,

$$k = \frac{0.5774 \pi E d}{\ln \frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$$

(8–20)

- Use Eq. (8–20) to find stiffness for each frustum
- Combine all frusta as springs in series

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots + \frac{1}{k_i}$$

(8–18)

Fig. 8–15b
Member Stiffness for Common Material in Grip

- If the grip consists of any number of members all of the same material, two identical frusta can be added in series. The entire joint can be handled with one equation,

\[
k_m = \frac{\pi E d \tan \alpha}{2 \ln \left( \frac{l \tan \alpha + d_w - d}{(l \tan \alpha + d_w + d) (d_w - d)} \right)}
\]  

(8–21)

- \(d_w\) is the washer face diameter
- Using standard washer face diameter of 1.5\(d\), and with \(\alpha = 30^\circ\),

\[
k_m = \frac{0.5774 \pi E d}{2 \ln \left( \frac{0.5774 l + 0.5d}{0.5774 l + 2.5d} \right)}
\]  

(8–22)
Finite Element Approach to Member Stiffness

- For the special case of common material within the grip, a finite element model agrees with the frustum model.
Finite Element Approach to Member Stiffness

- Exponential curve-fit of finite element results can be used for case of common material within the grip

\[
\frac{k_m}{Ed} = A \exp(Bd/l) \tag{8-23}
\]

<table>
<thead>
<tr>
<th>Material Used</th>
<th>Poisson Ratio</th>
<th>Elastic GPa</th>
<th>Modulus Mpsi</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.291</td>
<td>207</td>
<td>30.0</td>
<td>0.78715</td>
<td>0.62873</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.334</td>
<td>71</td>
<td>10.3</td>
<td>0.79670</td>
<td>0.63816</td>
</tr>
<tr>
<td>Copper</td>
<td>0.326</td>
<td>119</td>
<td>17.3</td>
<td>0.79568</td>
<td>0.63553</td>
</tr>
<tr>
<td>Gray cast iron</td>
<td>0.211</td>
<td>100</td>
<td>14.5</td>
<td>0.77871</td>
<td>0.61616</td>
</tr>
<tr>
<td>General expression</td>
<td></td>
<td></td>
<td></td>
<td>0.78952</td>
<td>0.62914</td>
</tr>
</tbody>
</table>

Note: Entire joint is made up of the same material

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*Table 8-8*

Stiffness Parameters of Various Member Materials

Example 8–2

As shown in Fig. 8–17a, two plates are clamped by washer-faced \( \frac{1}{2} \) in-20 UNF \( \times 1 \frac{1}{2} \) in SAE grade 5 bolts each with a standard \( \frac{1}{2} \) N steel plain washer.

(a) Determine the member spring rate \( k_m \) if the top plate is steel and the bottom plate is gray cast iron.

(b) Using the method of conical frusta, determine the member spring rate \( k_m \) if both plates are steel.

(c) Using Eq. (8–23), determine the member spring rate \( k_m \) if both plates are steel. Compare the results with part (b).

(d) Determine the bolt spring rate \( k_b \).
Example 8–2 (continued)

From Table A–32, the thickness of a standard $\frac{1}{2}$ N plain washer is 0.095 in. (a) As shown in Fig. 8–17b, the frusta extend halfway into the joint the distance

$$\frac{1}{2}(0.5 + 0.75 + 0.095) = 0.6725\text{ in}$$
Example 8–2 (continued)

The distance between the joint line and the dotted frusta line is $0.6725 - 0.5 - 0.095 = 0.0775$ in. Thus, the top frusta consist of the steel washer, steel plate, and 0.0775 in of the cast iron. Since the washer and top plate are both steel with $E = 30(10^6)$ psi, they can be considered a single frustum of 0.595 in thick. The outer diameter of the frustum of the steel member at the joint interface is $0.75 + 2(0.595) \tan 30^\circ = 1.437$ in. The outer diameter at the midpoint of the entire joint is $0.75 + 2(0.6725) \tan 30^\circ = 1.527$ in. Using Eq. (8–20), the spring rate of the steel is

$$k_1 = \frac{0.5774\pi (30)(10^6)0.5}{\ln \left\{ \frac{1.155(0.595) + 0.75 - 0.5)(0.75 + 0.5)}{[1.155(0.595) + 0.75 + 0.5)(0.75 - 0.5)} \right\}} = 30.80(10^6) \text{ lbf/in}$$

![Fig. 8–17b](Shigley's Mechanical Engineering Design)
Example 8–2 (continued)

From Tables 8–8 or A–5, for gray cast iron, \( E = 14.5 \) Mpsi. Thus for the upper cast-iron frustum

\[
k_2 = \frac{0.5774\pi(14.5)(10^6)0.5}{\ln\left\{ \frac{[1.155(0.0775) + 1.437 - 0.5](1.437 + 0.5)}{[1.155(0.0775) + 1.437 + 0.5](1.437 - 0.5)} \right\}} = 285.5(10^6) \text{ lbf/in}
\]

Fig. 8–17b
For the lower cast-iron frustum

\[ k_3 = \frac{0.5774\pi (14.5) (10^6) 0.5}{\ln \left\{ \frac{[1.155(0.6725) + 0.75 - 0.5](0.75 + 0.5)}{[1.155(0.6725) + 0.75 + 0.5](0.75 - 0.5)} \right\} } = 14.15(10^6) \text{ lbf/in} \]

Fig. 8–17b
Example 8–2 (continued)

The three frusta are in series, so from Eq. (8–18)

\[
\frac{1}{k_m} = \frac{1}{30.80 \times 10^6} + \frac{1}{285.5 \times 10^6} + \frac{1}{14.15 \times 10^6}
\]

This results in \( k_m = 9.378 \times 10^6 \) lb/in.  \textbf{Answer}

![Diagram of frusta](image)
Example 8–2 (continued)

(b) If the entire joint is steel, Eq. (8–22) with \( l = 2(0.6725) = 1.345 \) in gives

\[
k_m = \frac{0.5774\pi(30.0)(10^6)0.5}{2 \ln \left\{ 5 \left[ \frac{0.5774(1.345) + 0.5(0.5)}{0.5774(1.345) + 2.5(0.5)} \right] \right\} } \]

\( = 14.64(10^6) \) lbf/in.  \text{ Answer}

(c) From Table 8–8, \( A = 0.78715, B = 0.62873 \). Equation (8–23) gives

\[
k_m = 30(10^6)(0.5)(0.78715) \exp[0.62873(0.5)/1.345] = 14.92(10^6) \text{ lbf/in} \quad \text{Answer}
\]

For this case, the difference between the results for Eqs. (8–22) and (8–23) is less than 2 percent.

Fig. 8–17b
(d) Following the procedure of Table 8–7, the threaded length of a 0.5-in bolt is 
\[ L_T = 2(0.5) + 0.25 = 1.25 \text{ in}. \]
The length of the unthreaded portion is 
\[ l_d = 1.5 - 1.25 = 0.25 \text{ in}. \]
The length of the unthreaded portion in grip is 
\[ l_t = 1.345 - 0.25 = 1.095 \text{ in}. \]
The major diameter area is 
\[ A_d = \left(\frac{\pi}{4}\right)(0.5^2) = 0.1963 \text{ in}^2. \]
From Table 8–2, the tensile-stress area is 
\[ A_t = 0.1599 \text{ in}^2. \]
From Eq. (8–17)

\[
k_b = \frac{0.1963(0.1599)30(10^6)}{0.1963(1.095) + 0.1599(0.25)} = 3.69(10^6) \text{ lbf/in} \]

Answer

Fig. 8–17a
Bolt Materials

- Grades specify material, heat treatment, strengths
  - Table 8–9 for SAE grades
  - Table 8–10 for ASTM designations
  - Table 8–11 for metric property class
- Grades should be marked on head of bolt
**Bolt Materials**

- **Proof load** is the maximum load that a bolt can withstand without acquiring a permanent set.
- **Proof strength** is the quotient of proof load and tensile-stress area:
  - Corresponds to proportional limit
  - Slightly lower than yield strength
  - Typically used for static strength of bolt
- Good bolt materials have stress-strain curve that continues to rise to fracture.

![Stress-Strain Curve](image)
Tension Loaded Bolted Joints

\[ F_i = \text{preload} \]

\[ P_{\text{total}} = \text{Total external tensile load applied to the joint} \]

\[ P = \text{external tensile load per bolt} \]

\[ P_b = \text{portion of } P \text{ taken by bolt} \]

\[ P_m = \text{portion of } P \text{ taken by members} \]

\[ F_b = P_b + F_i = \text{resultant bolt load} \]

\[ F_m = P_m - F_i = \text{resultant load on members} \]

\[ C = \text{fraction of external load } P \text{ carried by bolt} \]

\[ 1 - C = \text{fraction of external load } P \text{ carried by members} \]

\[ N = \text{Number of bolts in the joint} \]
Tension Loaded Bolted Joints

- During bolt preload
  - bolt is stretched
  - members in grip are compressed
- When external load $P$ is applied
  - Bolt stretches an additional amount $\delta$
  - Members in grip uncompress same amount $\delta$

\[
\delta = \frac{P_b}{k_b} \quad \text{and} \quad \delta = \frac{P_m}{k_m}
\]

\[
P_m = P_b \frac{k_m}{k_b}
\]
Since \( P = P_b + P_m \),

\[
P_b = \frac{k_b P}{k_b + k_m} = CP
\]

\[
P_m = P - P_b = (1 - C)P
\]

- \( C \) is defined as the **stiffness constant** of the joint

\[
C = \frac{k_b}{k_b + k_m}
\]

- \( C \) indicates the proportion of external load \( P \) that the bolt will carry. A good design target is around 0.2.

### Table 8-12

<table>
<thead>
<tr>
<th>Bolt Grip, in</th>
<th>( k_b )</th>
<th>( k_m )</th>
<th>( C )</th>
<th>1 – ( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.57</td>
<td>12.69</td>
<td>0.168</td>
<td>0.832</td>
</tr>
<tr>
<td>3</td>
<td>1.79</td>
<td>11.33</td>
<td>0.136</td>
<td>0.864</td>
</tr>
<tr>
<td>4</td>
<td>1.37</td>
<td>10.63</td>
<td>0.114</td>
<td>0.886</td>
</tr>
</tbody>
</table>
Bolt and Member Loads

- The resultant bolt load is

\[ F_b = P_b + F_i = C \, P + F_i \quad F_m < 0 \]  \hfill (8-24)

- The resultant load on the members is

\[ F_m = P_m - F_i = (1 - C) \, P - F_i \quad F_m < 0 \]  \hfill (8-25)

- These results are only valid if the load on the members remains negative, indicating the members stay in compression.
Relating Bolt Torque to Bolt Tension

- Best way to measure bolt preload is by relating measured bolt elongation and calculated stiffness
- Usually, measuring bolt elongation is not practical
- Measuring applied torque is common, using a torque wrench
- Need to find relation between applied torque and bolt preload
Relating Bolt Torque to Bolt Tension

- From the power screw equations, Eqs. (8–5) and (8–6), we get

\[ T = \frac{F_id_m}{2} \left( \frac{l + \pi f d_m \sec \alpha}{\pi d_m - f l \sec \alpha} \right) + \frac{F_if_c d_c}{2} \]  

(a)

- Applying \( \tan \lambda = l/\pi d_m \),

\[ T = \frac{F_id_m}{2} \left( \frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + \frac{F_if_c d_c}{2} \]  

(b)

- Assuming a washer face diameter of 1.5\( d \), the collar diameter is \( d_c = (d + 1.5d)/2 = 1.25d \), giving

\[ T = \left[ \left( \frac{d_m}{2d} \right) \left( \frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + 0.625f_c \right] F_id \]  

(c)
Relating Bolt Torque to Bolt Tension

\[ T = \left[ \left( \frac{d_m}{2d} \right) \left( \frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + 0.625 f_c \right] F_i d \]  

- Define term in brackets as *torque coefficient* \( K \)

\[ K = \left( \frac{d_m}{2d} \right) \left( \frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + 0.625 f_c \]  

\[ T = K F_i d \]
Typical Values for Torque Coefficient $K$

$$T = K F_i d$$ \hspace{1cm} (8–27)

- Some recommended values for $K$ for various bolt finishes is given in Table 8–15
- Use $K = 0.2$ for other cases

**Table 8–15**

<table>
<thead>
<tr>
<th>Bolt Condition</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonplated, black finish</td>
<td>0.30</td>
</tr>
<tr>
<td>Zinc-plated</td>
<td>0.20</td>
</tr>
<tr>
<td>Lubricated</td>
<td>0.18</td>
</tr>
<tr>
<td>Cadmium-plated</td>
<td>0.16</td>
</tr>
<tr>
<td>With Bowman Anti-Seize</td>
<td>0.12</td>
</tr>
<tr>
<td>With Bowman-Grip nuts</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Distribution of Preload vs Torque

- Measured preloads for 20 tests at same torque have considerable variation
  - Mean value of 34.3 kN
  - Standard deviation of 4.91

Table 8–13

| 23.6, 27.6, 28.0, 29.4, 30.3, 30.7, 32.9, 33.8, 33.8, 33.8, |
| 34.7, 35.6, 35.6, 37.4, 37.8, 37.8, 39.2, 40.0, 40.5, 42.7 |

Mean value $\bar{F}_i = 34.3$ kN. Standard deviation, $\hat{\sigma} = 4.91$ kN.
Distribution of Preload vs Torque

- Same test with *lubricated* bolts
  - Mean value of 34.18 kN (unlubricated 34.3 kN)
  - Standard deviation of 2.88 kN (unlubricated 4.91 kN)

Table 8–14

| 30.3, 32.5, 32.5, 32.9, 32.9, 33.8, 34.3, 34.7, 37.4, 40.5 |

Mean value, $\bar{F}_i = 34.18$ kN. Standard deviation, $\hat{\sigma} = 2.88$ kN.

- Lubrication made little change to average preload vs torque
- Lubrication significantly reduces the standard deviation of preload vs torque
Example 8–3

A $\frac{3}{4}$ in-16 UNF $\times 2\frac{1}{2}$ in SAE grade 5 bolt is subjected to a load $P$ of 6 kip in a tension joint. The initial bolt tension is $F_i = 25$ kip. The bolt and joint stiffnesses are $k_b = 6.50$ and $k_m = 13.8$ Mlbf/in, respectively.

(a) Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.
(b) Specify the torque necessary to develop the preload, using Eq. (8–27).
(c) Specify the torque necessary to develop the preload, using Eq. (8–26) with $f = f_c = 0.15$. 
Example 8–3 (continued)

From Table 8–2, $A_t = 0.373$ in$^2$.

(a) The preload stress is

$$\sigma_i = \frac{F_i}{A_t} = \frac{25}{0.373} = 67.02 \text{ kpsi}$$

The stiffness constant is

$$C = \frac{k_b}{k_b + k_m} = \frac{6.5}{6.5 + 13.8} = 0.320$$

From Eq. (8–24), the stress under the service load is

$$\sigma_b = \frac{F_b}{A_t} = \frac{C \cdot P + F_i}{A_t} = C \cdot \frac{P}{A_t} + \sigma_i$$

$$= 0.320 \cdot \frac{6}{0.373} + 67.02 = 72.17 \text{ kpsi}$$

From Table 8–9, the SAE minimum proof strength of the bolt is $S_p = 85$ kpsi. The preload and service load stresses are respectively 21 and 15 percent less than the proof strength.
Example 8–3 (continued)

(b) From Eq. (8–27), the torque necessary to achieve the preload is

\[ T = K F_i d = 0.2(25)(10^3)(0.75) = 3750 \text{ lbf} \cdot \text{in} \]

Answer

(c) The minor diameter can be determined from the minor area in Table 8–2. Thus

\[ d_r = \sqrt{\frac{4A_r}{\pi}} = \sqrt{4(0.351)/\pi} = 0.6685 \text{ in.} \]

Thus, the mean diameter is

\[ d_m = \frac{0.75 + 0.6685}{2} = 0.7093 \text{ in.} \]

The lead angle is

\[ \lambda = \tan^{-1} \left( \frac{l}{\pi d_m} \right) = \tan^{-1} \left( \frac{1}{\pi d_m N} \right) = \tan^{-1} \left( \frac{1}{\pi (0.7093)(16)} \right) = 1.6066^\circ \]

For \( \alpha = 30^\circ \), Eq. (8–26) gives

\[
T = \left\{ \left[ \frac{0.7093}{2(0.75)} \right] \left[ \frac{\tan 1.6066^\circ + 0.15(\sec 30^\circ)}{1 - 0.15(\tan 1.6066^\circ)(\sec 30^\circ)} \right] + 0.625(0.15) \right\} 25(10^3)(0.75)
\]

\[ = 3551 \text{ lbf} \cdot \text{in} \]

which is 5.3 percent less than the value found in part (b).
Bolt and Member Loads

- The resultant bolt load is
  \[ F_b = P_b + F_i = C \, P + F_i \quad F_m < 0 \]  
  \[ \text{(8–24)} \]

- The resultant load on the members is
  \[ F_m = P_m - F_i = (1 - C) \, P - F_i \quad F_m < 0 \]  
  \[ \text{(8–25)} \]

- These results are only valid if the load on the members remains negative, indicating the members stay in compression.
Tension Loaded Bolted Joints: Static Factors of Safety

Axial Stress:

\[ \sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t} \]

Yielding Factor of Safety:

\[ n_p = \frac{S_p}{\sigma_b} = \frac{S_p}{(CP + F_i)/A_t} = \frac{S_p A_t}{CP + F_i} \quad (8-28) \]

Load Factor:

\[ \frac{C n_L P + F_i}{A_t} = S_p \quad n_L = \frac{S_p A_t - F_i}{CP} \quad (8-29) \]

Joint Separation Factor:

\[ n_0 = \frac{F_i}{P (1 - C)} \quad (8-30) \]
Recommended Preload

\[ F_i = \begin{cases} 
0.75F_p & \text{for nonpermanent connections, reused fasteners} \\
0.90F_p & \text{for permanent connections}
\end{cases} \quad (8-31) \]

\[ F_p = A_t S_p \quad (8-32) \]
Figure 8–19 is a cross section of a grade 25 cast-iron pressure vessel. A total of $N$ bolts are to be used to resist a separating force of 36 kip.

(a) Determine $k_b$, $k_m$, and $C$.

(b) Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.

(c) With the number of bolts obtained in part (b), determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.
Example 8–4 (continued)

(a) The grip is \( l = 1.50 \) in. From Table A–31, the nut thickness is \( \frac{35}{64} \) in. Adding two threads beyond the nut of \( \frac{2}{11} \) in gives a bolt length of

\[
L = \frac{35}{64} + 1.50 + \frac{2}{11} = 2.229 \text{ in}
\]

From Table A–17 the next fraction size bolt is \( L = 2\frac{1}{4} \) in. From Eq. (8–13), the thread length is \( L_T = 2(0.625) + 0.25 = 1.50 \) in. Thus, the length of the unthreaded portion in the grip is \( l_d = 2.25 - 1.50 = 0.75 \) in. The threaded length in the grip is \( l_t = l - l_d = 0.75 \) in. From Table 8–2, \( A_t = 0.226 \) in\(^2\). The major-diameter area is \( A_d = \pi (0.625)^2 / 4 = 0.3068 \) in\(^2\). The bolt stiffness is then

\[
k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.3068(0.226)(30)}{0.3068(0.75) + 0.226(0.75)} = 5.21 \text{ Mlbf/in}
\]

Answer
Example 8–4 (continued)

From Table A–24, for no. 25 cast iron we will use \( E = 14 \) Mpsi. The stiffness of the members, from Eq. (8–22), is

\[
 k_m = \frac{0.5774\pi E d}{2 \ln \left( \frac{0.5774l + 0.5d}{0.5774l + 2.5d} \right)} = \frac{0.5774\pi (14)(0.625)}{2 \ln \left[ \frac{0.5774 (1.5) + 0.5 (0.625)}{0.5774 (1.5) + 2.5 (0.625)} \right]}
\]

\[= 8.95 \text{ Mlbf/in} \quad \text{Answer}\]

If you are using Eq. (8–23), from Table 8–8, \( A = 0.77871 \) and \( B = 0.61616 \), and

\[
k_m = E d A \exp(Bd/l)
\]

\[= 14(0.625)(0.77871) \exp[0.61616(0.625)/1.5]
\]

\[= 8.81 \text{ Mlbf/in} \]

which is only 1.6 percent lower than the previous result.

From the first calculation for \( k_m \), the stiffness constant \( C \) is

\[
 C = \frac{k_b}{k_b + k_m} = \frac{5.21}{5.21 + 8.95} = 0.368 \quad \text{Answer}
\]
Example 8–4 (continued)

(b) From Table 8–9, \( S_p = 85 \text{ ksi} \). Then, using Eqs. (8–31) and (8–32), we find the recommended preload to be

\[
F_i = 0.75 A_t S_p = 0.75(0.226)(85) = 14.4 \text{ kip}
\]

For \( N \) bolts, Eq. (8–29) can be written

\[
n_L = \frac{S_p A_t - F_i}{C (P_{\text{total}}/N)}
\]

or

\[
N = \frac{C n_L P_{\text{total}}}{S_p A_t - F_i} = \frac{0.368(2)(36)}{85(0.226) - 14.4} = 5.52
\]

Six bolts should be used to provide the specified load factor.
Example 8–4 (continued)

(c) With six bolts, the load factor actually realized is

\[ n_L = \frac{85(0.226) - 14.4}{0.368(36/6)} = 2.18 \]

From Eq. (8–28), the yielding factor of safety is

\[ n_p = \frac{S_p A_t}{C (P_{\text{total}}/N) + F_i} = \frac{85(0.226)}{0.368(36/6) + 14.4} = 1.16 \]

From Eq. (8–30), the load factor guarding against joint separation is

\[ n_0 = \frac{F_i}{(P_{\text{total}}/N)(1 - C)} = \frac{14.4}{(36/6)(1 - 0.368)} = 3.80 \]
Tension Loaded Bolted Joints: Static Factors of Safety

Axial Stress:

\[ \sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t} \]

Yielding Factor of Safety:

\[ n_p = \frac{S_p}{\sigma_b} = \frac{S_p}{(CP + F_i)/A_t} = \frac{S_p A_t}{CP + F_i} \quad (8-28) \]

Load Factor:

\[ \frac{C n_L P + F_i}{A_t} = S_p \]

\[ n_L = \frac{S_p A_t - F_i}{CP} \quad (8-29) \]

Joint Separation Factor:

\[ n_0 = \frac{F_i}{P (1 - C)} \quad (8-30) \]
Fatigue Loading of Tension Joints

- Fatigue methods of Ch. 6 are directly applicable
- Distribution of typical bolt failures is
  - 15% under the head
  - 20% at the end of the thread
  - 65% in the thread at the nut face
- Fatigue stress-concentration factors for threads and fillet are given in Table 8–16

**Table 8–16**

<table>
<thead>
<tr>
<th>Fatigue Stress-Concentration Factors $K_f$ for Threaded Elements</th>
<th>SAE Grade</th>
<th>Metric Grade</th>
<th>Rolled Threads</th>
<th>Cut Threads</th>
<th>Fillet</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 2</td>
<td>3.6 to 5.8</td>
<td>2.2</td>
<td>2.8</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>4 to 8</td>
<td>6.6 to 10.9</td>
<td>3.0</td>
<td>3.8</td>
<td>2.3</td>
<td></td>
</tr>
</tbody>
</table>
Endurance Strength for Bolts

- Bolts are standardized, so endurance strengths are known by experimentation, including all modifiers discussed in chapter 6. See Table 8–17.
- Fatigue stress-concentration factor $K_f$ should not be applied to the nominal bolt stresses.
- Ch. 6 methods can be used for cut threads.

<table>
<thead>
<tr>
<th>Table 8–17</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grade or Class</strong></td>
</tr>
<tr>
<td>Fully Corrected</td>
</tr>
<tr>
<td>Endurance Strengths for</td>
</tr>
<tr>
<td>Bolts and Screws with</td>
</tr>
<tr>
<td>Rolled Threads*</td>
</tr>
<tr>
<td>ISO 8.8</td>
</tr>
<tr>
<td>ISO 9.8</td>
</tr>
<tr>
<td>ISO 10.9</td>
</tr>
<tr>
<td>ISO 12.9</td>
</tr>
</tbody>
</table>

*Repeatedly applied, axial loading, fully corrected.
Fatigue Stresses

With an external load on a per bolt basis fluctuating between $P_{\text{min}}$ and $P_{\text{max}}$,

\begin{align}
F_{b_{\text{min}}} &= CP_{\text{min}} + F_i \\
F_{b_{\text{max}}} &= CP_{\text{max}} + F_i
\end{align}

\begin{align}
\sigma_a &= \frac{(F_{b_{\text{max}}} - F_{b_{\text{min}}})/2}{A_t} = \frac{(CP_{\text{max}} + F_i) - (CP_{\text{min}} + F_i)}{2A_t} \\
\sigma_a &= \frac{C(P_{\text{max}} - P_{\text{min}})}{2A_t}
\end{align}

\begin{align}
\sigma_m &= \frac{(F_{b_{\text{max}}} + F_{b_{\text{min}}})/2}{A_t} = \frac{(CP_{\text{max}} + F_i) + (CP_{\text{min}} + F_i)}{2A_t} \\
\sigma_m &= \frac{C(P_{\text{max}} + P_{\text{min}})}{2A_t} + \frac{F_i}{A_t}
\end{align}
Yield Check with Fatigue Stresses

- As always, maximum stress must be checked for static yielding, using $S_p$ instead of $S_y$.
- In fatigue loading situations, since $\sigma_a$ and $\sigma_m$ are already calculated, it may be convenient to check yielding with

\[ n_p = \frac{S_p}{\sigma_m + \sigma_a} \]  

(8–51)

- This is equivalent to the yielding factor of safety from Eq. (8–28).

\[ n_p = \frac{S_p}{\sigma_b} = \frac{S_p}{(CP + F_i)/A_t} = \frac{S_p A_t}{CP + F_i} \]  

(8–28)
Fatigue Factor of Safety

- Fatigue factor of safety based on Goodman line and constant preload load line,

\[ n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} \]  (8–38)

- Other failure theories can be used, following the same approach.
Repeated Load Special Case

- Fatigue factor of safety equations for repeated loading, constant preload load line, with various failure curves:
  
  **Goodman:**
  
  \[
  n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)}
  \tag{8-45}
  \]

  **Gerber:**
  
  \[
  n_f = \frac{1}{2\sigma_a S_e} \left[ S_{ut} \sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e \right]
  \tag{8-46}
  \]

  **ASME-elliptic:**
  
  \[
  n_f = \frac{S_e}{\sigma_a(S_p^2 + S_e^2)} \left( S_p \sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e \right)
  \tag{8-47}
  \]