Gears and Gearing
Part 1
Types of Gears

- Spur Gears
- Helical Gears
- Spiral Bevel Gears
- Spur Bevel Gears
- Spiral Bevel Gears
- Pinion Gear
- Ring Gear
- Rack Gear
- Rack-and-Pinion Gears
- Worm Gears
- Internal Gear
- Planet Pinion
- Sun Gear and Shaft
- Planet-Pinion Carrier and Shaft
- Hypoid Gears
Types of Gears

- **Spur**
- **Helical**
- **Bevel**
- **Worm**
Nomenclature of Spur-Gear Teeth

- Addendum
- Dedendum
- Circumference pitch
- Tooth thickness
- Clearance
- Fillet radius
- Dedendum circle
- Clearance circle
- Pitch circle
- Width of space
- Face
- Flank
- Top land
- Bottom land

Fig. 13–5
Tooth Size, Diameter, Number of Teeth

\[ P = \frac{N}{d} \]  \hspace{1cm} (13-1)

\[ m = \frac{d}{N} \]  \hspace{1cm} (13-2)

\[ p = \frac{\pi d}{N} = \pi m \]  \hspace{1cm} (13-3)

\[ pp = \pi \]  \hspace{1cm} (13-4)

where

- \( P \) = diametral pitch, teeth per inch
- \( N \) = number of teeth
- \( d \) = pitch diameter, in or mm
- \( m \) = module, mm
- \( p \) = circular pitch, in or mm
## Tooth Sizes in General Industrial Use

<table>
<thead>
<tr>
<th>Diametral Pitch</th>
<th>Coarse</th>
<th>2, 2(\frac{1}{4}), 2(\frac{1}{2}), 3, 4, 6, 8, 10, 12, 16</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fine</td>
<td>20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modules</th>
<th>Preferred</th>
<th>1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Next Choice</td>
<td>1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45</td>
</tr>
</tbody>
</table>

Table 13–2
Standardized Tooth Systems

- Common pressure angles $\phi$: 20° and 25°
- Older pressure angle: 14 ½°
- Common face width:

$$3p < F < 5p$$

$$p = \frac{\pi}{P}$$

$$\frac{3\pi}{P} < F < \frac{5\pi}{P}$$
Gear Sources

- Boston Gear
- Martin Sprocket
- W. M. Berg
- Stock Drive Products

... Numerous others
Conjugate Action

- When surfaces roll/slide against each other and produce constant angular velocity ratio, they are said to have *conjugate action*.
- Can be accomplished if instant center of velocity between the two bodies remains stationary between the grounded instant centers.
Fundamental Law of Gearing

Fundamental law of gearing:
The common normal of the tooth profiles at all points within the mesh must always pass through a fixed point on the line of the centers called pitch point. Then the gearset’s velocity ratio will be constant through the mesh and be equal to the ratio of the gear radii.
Conjugate Action: Fundamental Law of Gearing

- Forces are transmitted on line of action which is normal to the contacting surfaces.
- Velocity. $V_P$ of both gears is the same at point $P$, the pitch point.
- Angular velocity ratio is inversely proportional to the radii to point $P$, the pitch point.

$$\left| \frac{\omega_1}{\omega_2} \right| = \frac{r_2}{r_1} \quad (13-5)$$

- Circles drawn through $P$ from each fixed pivot are pitch circles, each with a pitch radius.
Gear Ratio

\( V_P \) of both gears is the same at point \( P \), the *pitch (circle contact) point*

\[
V_P = \omega_1 r_1 = \omega_2 r_2
\]

Pitch Circle of Gears

\[
\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1}
\]

\( \omega_2 \) rotates opposite of \( \omega_1 \)

Gear Ratio \( \geq 1 \)
Nomenclature

Smaller Gear is Pinion and Larger one is the gear

In most application the pinion is the driver, This reduces speed but it increases torque.
Simple Gear Trains

For a pinion 2 driving a gear 3, the speed of the driven gear is

\[ n_3 = \left| \frac{N_2}{N_3} n_2 \right| = \left| \frac{d_2}{d_3} n_2 \right| \]

where \( n = \) revolutions or rev/min

\( N = \) number of teeth

\( d = \) pitch diameter
Simple Train Value

\[
n_6 = -\frac{N_2 \ N_3 \ N_5}{N_3 \ N_4 \ N_6} n_2
\]

\[
e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}
\]

\[
n_L = e n_F
\]
Compound Gear Train

- A practical limit on train value for one pair of gears is 10 to 1
- To obtain more, compound two gears onto the same shaft

\[ e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}} \]
Compound Gear Trains

\[ \frac{n_5}{n_1} = \left( -\frac{N_1}{N_2} \right) \left( -\frac{N_3}{N_4} \right) \left( -\frac{N_4}{N_5} \right) \]
Example 13–3

A gearbox is needed to provide a 30:1 (± 1 percent) increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

Solution

Since the ratio is greater than 10:1, but less than 100:1, a two-stage compound gear train, such as in Figure 13–28, is needed. The portion to be accomplished in each stage is $\sqrt{30} = 5.4772$. For this ratio, assuming a typical 20° pressure angle, the minimum number of teeth to avoid interference is 16, according to Eq. (13–11). The number of teeth necessary for the mating gears is

$$16\sqrt{30} = 87.64 \div 88$$

From Eq. (13–30), the overall train value is

$$e = \frac{88}{16}(\frac{88}{16}) = 30.25$$

This is within the 1 percent tolerance. If a closer tolerance is desired, then increase the pinion size to the next integer and try again.
Example 13–4

A gearbox is needed to provide an *exact* 30:1 increase in speed, while minimizing the overall gearbox size. Specify appropriate teeth numbers.

**Solution**

The previous example demonstrated the difficulty with finding integer numbers of teeth to provide an exact ratio. In order to obtain integers, factor the overall ratio into two integer stages.

\[ e = 30 = (6)(5) \]

\[ N_2/N_3 = 6 \quad \text{and} \quad N_4/N_5 = 5 \]

With two equations and four unknown numbers of teeth, two free choices are available. Choose \( N_3 \) and \( N_5 \) to be as small as possible without interference. Assuming a 20° pressure angle, Eq. (13–11) gives the minimum as 16.
Example 13–4

Then

\[ N_2 = 6 \quad N_3 = 6 \times (16) = 96 \]
\[ N_4 = 5 \quad N_5 = 5 \times (16) = 80 \]

The overall train value is then exact.

\[ e = \frac{96}{16} \times \frac{80}{16} = (6)(5) = 30 \]

Answer
Example 13–5

A gearbox is needed to provide an exact 30:1 increase in speed, while minimizing the overall gearbox size. The input and output shafts should be in-line. Specify appropriate teeth numbers.

**Solution**
The governing equations are

\[
\frac{N_2}{N_3} = 6 \\
\frac{N_4}{N_5} = 5 \\
N_2 + N_3 = N_4 + N_5
\]

With three equations and four unknown numbers of teeth, only one free choice is available. Of the two smaller gears, \(N_3\) and \(N_5\), the free choice should be used to minimize \(N_3\) since a greater gear ratio is to be achieved in this stage. To avoid interference, the minimum for \(N_3\) is 16.
Example 13–5

Applying the governing equations yields

\[ N_2 = 6N_3 = 6(16) = 96 \]
\[ N_2 + N_3 = 96 + 16 = 112 = N_4 + N_5 \]

Substituting \( N_4 = 5N_5 \) gives

\[ 112 = 5N_5 + N_5 = 6N_5 \]
\[ N_5 = 112/6 = 18.67 \]

If the train value need only be approximated, then this can be rounded to the nearest integer. But for an exact solution, it is necessary to choose the initial free choice for \( N_3 \) such that solution of the rest of the teeth numbers results exactly in integers. This can be done by trial and error, letting \( N_3 = 17 \), then 18, etc., until it works. Or, the problem can be normalized to quickly determine the minimum free choice. Beginning again, let the free choice be \( N_3 = 1 \). Applying the governing equations gives
Example 13–5

\[ N_2 = 6N_3 = 6(1) = 6 \]

\[ N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5 \]

Substituting \( N_4 = 5N_5 \), we find

\[ 7 = 5N_5 + N_5 = 6N_5 \]

\[ N_5 = 7/6 \]

This fraction could be eliminated if it were multiplied by a multiple of 6. The free choice for the smallest gear \( N_3 \) should be selected as a multiple of 6 that is greater than the minimum allowed to avoid interference. This would indicate that \( N_3 = 18 \).
Repeating the application of the governing equations for the final time yields

\[ N_2 = 6N_3 = 6(18) = 108 \]
\[ N_2 + N_3 = 108 + 18 = 126 = N_4 + N_5 \]
\[ 126 = 5N_5 + N_5 = 6N_5 \]
\[ N_5 = 126/6 = 21 \]
\[ N_4 = 5N_5 = 5(21) = 105 \]

Thus,

\[ N_2 = 108 \]
\[ N_3 = 18 \]
\[ N_4 = 105 \]
\[ N_5 = 21 \]  

Answer
Example 13–5

Checking, we calculate \( e = (108/18)(105/21) = (6)(5) = 30 \).
And checking the geometry constraint for the in-line requirement, we calculate

\[
N_2 + N_3 = N_4 + N_5 \\
108 + 18 = 105 + 21 \\
126 = 126
\]
The most common conjugate profile is the *involute* profile.

Can be generated by unwrapping a string from a cylinder, keeping the string taut and tangent to the cylinder.

Circle is called *base circle*.

**Fig. 13–8**
Involute Profile Producing Conjugate Action

Fig. 13–7
Circles of a Gear Layout

Fig. 13–9
Sequence of Gear Layout

- Pitch circles in contact
- Pressure line at desired pressure angle
- Base circles tangent to pressure line
- Involute profile from base circle
- Cap teeth at addendum circle at 1/P from pitch circle
- Root of teeth at dedendum circle at 1.25/P from pitch circle (clearance)
- Tooth spacing from circular pitch, \( p = \pi / P \)

Fig. 13–9