Digital Control Systems

Discretization & Digitization of an analog (continuous) signal

• Time sample
  – Usually uniform rate
  – new value each $\Delta T$

• Level of magnitude limited
  – 8-bit $=2^8=256$ levels
  – 0-5 volts at 8-bit resolution
    $5V/256= 0.0195313$ V
Digital (Discrete) Controllers

Set-up for a typical digital controlled system
Sampled values of continuous signals (in/out)

Physical system with controller, sensors, actuators

Continuous vs. Discrete Time
Continuous Controllers

- Differential equations
- All analog signals and devices
- Amplifiers, resistors, capacitors, inductors hardwired
- Knobs connected to potentiometers for input and interfaces
- Not very easy to manipulate and display

Digital/Discrete Systems

- Difference equations
- Use digital (computer) devices for control and signal processing
- Very flexible to change control
- Easy to manipulate and present Inputs/Outputs via computer display
- Still need to analog circuits for power to actuator and some output sensor
Pros and Cons of Digital Signal Processing

- **Pros**
  - Accuracy can be controlled by choosing word length, number of bits
  - Many systems inherently digital
  - Repeatable
  - Control algorithms easily modified
  - Sensitivity to electrical noise is minimal
  - Flexibility can be achieved with software implementations
  - Non-linear and time-varying operations are easier to implement
  - Digital storage is cheap
  - Price/performance and reduced time-to-market

- **Cons**
  - Sampling causes loss of information
  - A/D and D/A requires mixed-signal hardware
  - Limited speed of processors
  - Quantization and round-off errors
Sampling

Result of sampling a continuous time signal

The result of sampling

Digital Control Systems:
Zero-Order Hold (Sample and hold)
Sampling time ($\Delta T$)

- $\Delta T$ Limits
  - Based on CPU and ALL algorithms
  - Nyquist Frequency: Limit on highest system frequency that can be detected which is half the sampling frequency (Hz) $(2/\Delta T)$

Aliasing effect when using low sampling rate
Digital Control Systems: Sampling real signals: **Zero-Order Hold**

\[ H_2(s) = \frac{1 - e^{-sT}}{s} \]

![Diagram of Zero-Order Hold](image)

**Figure 12.1:** *Aliasing effect when using low sampling rate*

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12/7/2011
The Basic Control System

Implemented on a microcontroller …

The Basic Tasks

• Measure the process variable
• Compute control algorithm
  – data scaling
  – filtering
  – decision making
• Generate controller output signal

REPEAT
Timing

- Basic Tasks performed periodically

- Sampling frequency rule-of-thumb:
  5X-10X closed-loop bandwidth (Hz)

Z-transform meaning of ‘z’

\[ Z\{x(t)\} = x(0) + x(T)z^{-1} + x(2T)z^{-2} + x(3T)z^{-3} + ... + x(kT)z^{-k} \]
Digital Control Systems:
The z-transform

Def’n of z-Transform:

\[ Z\{f(t)\} = F(z) = \sum_{k=0}^{\infty} \left( f(kT) z^{-k} \right) \]

Relationship between s-plane and z-plane: a delay

\[ z = e^{sT} \]

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Digital Control Systems:
The z-transform (cont)

Several common z-Transforms

<table>
<thead>
<tr>
<th>x(t)</th>
<th>X(s)</th>
<th>X(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u(t)</td>
<td>( \frac{1}{s} )</td>
<td>( \frac{z}{z-1} )</td>
</tr>
<tr>
<td>t</td>
<td>( \frac{1}{s^2} )</td>
<td>( \frac{Tz}{(z-1)^2} )</td>
</tr>
<tr>
<td>e^{-at}</td>
<td>( \frac{1}{s+a} )</td>
<td>( \frac{z}{z-e^{-aT}} )</td>
</tr>
<tr>
<td>\sin(\omega t)</td>
<td>( \frac{\omega}{s^2 + \omega^2} )</td>
<td>( \frac{z \sin(\omega T)}{z^2 - 2\cos(\omega T) + 1} )</td>
</tr>
</tbody>
</table>
Digital Control Systems: 
Stability in s-plane

Region of Stability: 
Left-Hand Plane

Digital Control Systems: 
Stability in z-plane 
Region of Stability: 
Unit-Circle
Digital Control Systems: Root-Locus

\[ Y(z) = \frac{K_G(z) D(z)}{R(z)} \]

\[ 1 + K_G(z) D(z) = 0 \]

Digital Control Systems: Root-Locus (cont)
Digital Control Systems: MATLAB to the rescue

• c2d  
  - conversion of continuous-time models to discrete time

• zgrid  
  - generate z-plane grid lines for a root locus or pole-zero map over an existing map

• dstep  
  - Step response of discrete time system

• stairs  
  - Connects the elements from dstep to form stairstep graph

Digital Control Systems:  
Design Example

Root Locus Design for Digital DC Motor Position Control
Digital Control Systems: Design Example

As CPU’s get faster discrete systems approach continuous.

A digital stability check is always recommended.