EVE 402
Air Pollution Generation and Control

Chapter #5 Lectures (Part 2)



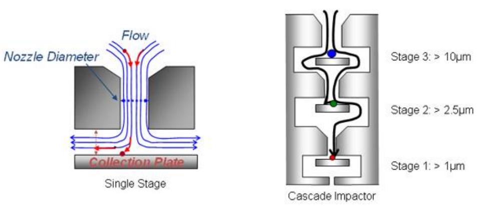
## How to Characterize Aerosols

- Size
  - Monodisperse: All particles are the same size
  - Polydisperse: Particles of more than one size
- Concentration
  - Number concentration by counting
  - Mass concentration by weight measurement
- Differential or Continuous Analysis
  - We assume that particle diameters are continuously distributed



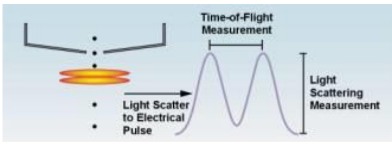
## Detection/Collection Methods

1. Cascade Impactor



# Detection/Collection Methods (2)

2. Aerodynamic Particle Sizer (APS)






# Example data from a cascade impactor

| Size Range (µm) | Count (#) | Fraction | Percent (%) | Cumulative Percent (%) | Fraction/size ($\mu m^{-1}$) |
|---|---|---|---|---|---|
| 0-4 | 104 | 0.104 | 10.4 | 10.4 | 0.026 |
| 4-6 | 160 | 0.16 | 16.0 | 26.4 | 0.08 |
| 6-8 | 161 | 0.161 | 16.1 | 42.5 | 0.0805 |
| 8-9 | 75 | 0.075 | 7.5 | 50.0 | 0.075 |
| 9-10 | 67 | 0.067 | 6.7 | 56.7 | 0.067 |
| 10-14 | 186 | 0.186 | 18.6 | 75.3 | 0.465 |
| 14-16 | 61 | 0.61 | 6.1 | 81.4 | 0.0305 |
| 16-20 | 79 | 0.79 | 7.9 | 89.3 | 0.0197 |
| 20-35 | 103 | 0.103 | 10.3 | 99.6 | 0.0034 |
| 35-50 | 4 | 0.004 | 0.4 | 100.0 | 0.0001 |
| > 50 | 0 | 0 | 0 | 100.0 | 0 |
| Total | 1000 | | 100.0 | | |

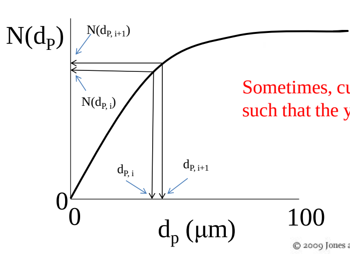


# Cumulative Particle Size Distributions

Let: N = total number of particles of all sizes

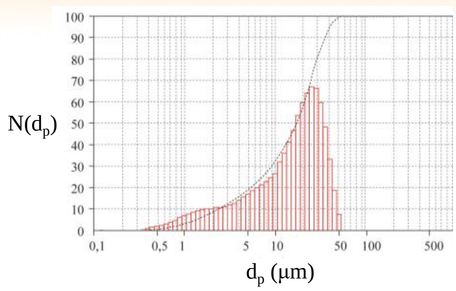
$N(d_p)$ = no. of particles with diam. ≤ $d_p$

= *cumulative number distribution*



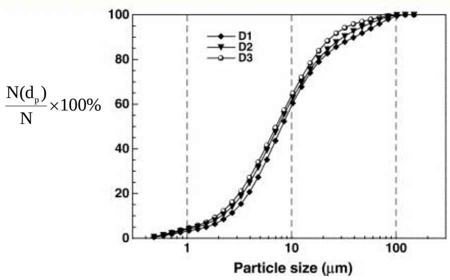

Sometimes, curve is normalized, such that the y-axis is $N(d_p)/N$

## A Typical Cumulative Number Distribution





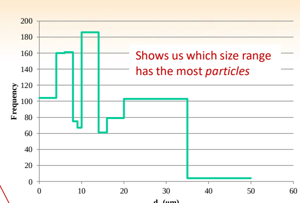

## A Typical *Normalized* Cumulative Number Distribution






## Histogram of Example Data

| Size Range (μm) | Count, n($d_p$) (#) |
|---|---|
| 0-4 | 104 |
| 4-6 | 160 |
| 6-8 | 161 |
| 8-9 | 75 |
| 9-10 | 67 |
| 10-14 | 186 |
| 14-16 | 61 |
| 16-20 | 79 |
| 20-35 | 103 |
| 35-50 | 4 |
| > 50 | 0 |
| Total | 1000 |

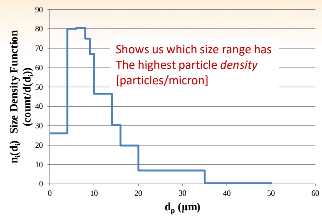



$$n(d_p) = \frac{dN(d_p)}{d(d_p)}$$

$n(d_p)$ is the number of particles per unit differential range of particle size.
That is, the number of particles per size range $d_p$ to $d_p + d(d_p)$

## Size Density Function

| Size range [µm] | $n(d_p)/d(d_p)$ [#/µm] |
|---|---|
| 0 – 4 | 26 |
| 4 – 6 | 80 |
| 6 – 8 | 80.5 |
| 8 – 9 | 75 |
| 9 – 10 | 67 |
| 10 – 14 | 46.5 |
| 14 – 16 | 30.5 |
| 16 – 20 | 19.25 |
| 20 – 35 | 6.87 |
| 35 – 50 | 0.27 |
| > 50 | 0 |

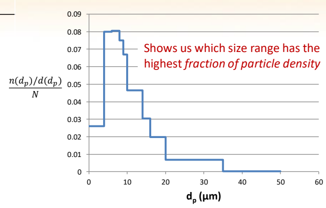



$$\text{Example: } \frac{n(d_p)}{d(d_p)} = \frac{\text{count}}{\Delta d_p} = \frac{104}{4\mu m} = 26\ \mu m^{-1}$$

 10

## Normalized Size Density Function

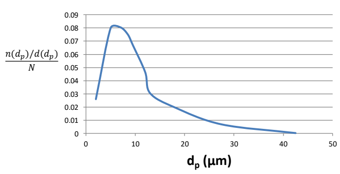
| Size range [µm] | fraction/size |
|---|---|
| 0 – 4 | 0.026 |
| 4 – 6 | 0.080 |
| 6 – 8 | 0.081 |
| 8 – 9 | 0.075 |
| 9 – 10 | 0.067 |
| 10 – 14 | 0.047 |
| 14 – 16 | 0.031 |
| 16 – 20 | 0.019 |
| 20 – 35 | 0.007 |
| 35 – 50 | 0.0003 |
| > 50 | 0 |




 11

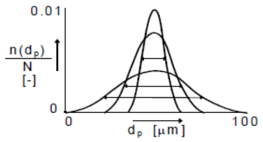
## Normalized Size Density Plot - Continuous (points are plotted to midpoint of size range)

| Size range [µm] | fraction/size |
|---|---|
| 0 – 4 | 0.026 |
| 4 – 6 | 0.080 |
| 6 – 8 | 0.081 |
| 8 – 9 | 0.075 |
| 9 – 10 | 0.067 |
| 10 – 14 | 0.047 |
| 14 – 16 | 0.031 |
| 16 – 20 | 0.019 |
| 20 – 35 | 0.007 |
| 35 – 50 | 0.0003 |
| > 50 | 0 |




 12

By inspection, what can you say about the Three different size density functions below?

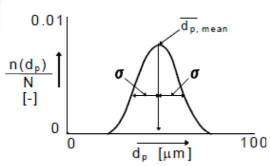



They all have the same ___________ $d_p$

They all have different ____________

 13

The schematic shows a symmetric distribution of particles and its corresponding $\bar{d}_{P,mean}$ and σ




$\sigma =$ standard deviation for a population

s = standard deviation of a sample

$$s = \left[ \frac{\sum_{i=1}^{\infty} \left( d_{pi} - \bar{d}_{p,mean} \right)^2 n\left(d_{pi}\right)}{N-1} \right]^{1/2}$$

$$\bar{d}_{p,mean} = \frac{\sum_{i=1}^{\infty} d_{pi}\, n\left(d_{pi}\right)}{N} \quad \text{(based on number)}$$

where:

$d_{pi} =$ characteristic size (diameter) of particles in a given size range

$n\left(d_{pi}\right) =$ number of particles within a characteristic size range

 14

## Gaussian Size Distribution

- The normal (Gaussian) distribution may be used to model particle size distributions.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[ -\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2 \right]$$

$$\frac{n\left(d_p\right)}{N} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[ -\frac{1}{2} \left( \frac{d_p - \bar{d}_{p,\,mean}}{\sigma} \right)^2 \right]$$

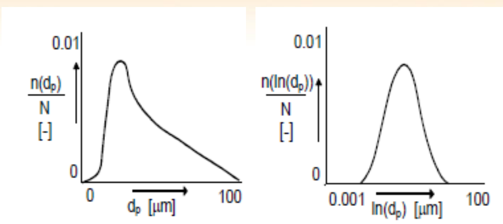
 15

## Gaussian Size Distribution (2)

- Normalized size density functions are often asymmetric
- Symmetry may be regained by using a transformation, such as $\log(d_p)$ or $\ln(d_p)$
- The transformation of $d_p$ may provide an equation that better fits the data than the normal distribution



**Asymmetric Distribution** | **Symmetric Distribution**






## Observations of atmospheric aerosol sizes show mostly log-normal distributions: $n(\ln(d_p))$

$$\frac{n(\ln(d_p))}{N} = \frac{1}{\ln(\sigma_g)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(d_p) - \ln(\bar{d}_{p,g})}{\ln(\sigma_g)}\right)^2\right]$$

## When using a log-normal distribution, there is interest in the

- Geometric mean diameter

$$\bar{d}_{p,g} = \exp\left[\frac{\sum_{i=1}^{\infty} n(d_{p_i})\ln(d_{p_i})}{N}\right]$$

- Geometric standard deviation

$$\sigma_g = \exp\left[\left(\frac{\sum_{i=1}^{\infty} n(d_{p_i})\left(\ln(d_{p_i}) - \ln(\bar{d}_{p,g})\right)^2}{N-1}\right)^{1/2}\right]$$





## What is the $\bar{d}_{p,g}$?

- In general, the geometric mean indicates the central tendency of a set using the *product* (the arithmetic mean uses the *sum*)
  - Say we have four $d_p$ values [μm]: 4.5, 24.6, 13.2, 19.7
    - $\bar{d}_p$ = (4.5+24.6+13.2+19.7)/4 = 15.5 μm
    - $\bar{d}_{p,g}$ = $\sqrt[4]{4.5 \times 24.6 \times 13.2 \times 19.7} = 13.03$ μm
- In terms of logarithms, we compute the arithmetic mean of the transformed data and use exponentiation to return to original scale
  - $\bar{d}_{p,g} = exp\left[\frac{1}{N}\sum_{i=1}^{N}\ln(a_i)\right]$




## $\bar{d}_{p,g}$ (logarithms)

- So, from the example data
  - $\bar{d}_{p,g} = \exp\left[\frac{1}{4}(\ln 4.5 + \ln 24.6 + \ln 13.2 + \ln 19.7)\right]$
  - ∴ $\bar{d}_{p,g}$ = 13.03 μm
- Note…this also works for $\log_{10}$, or any other base

## What is the $\sigma_g$?

- In terms of logarithms (ln), $\sigma_g$ is the exponentiated result of the standard deviation of the transformed values
- So, since the transformed data points are: 1.5, 3.2, 2.58, 2.98
  - $s_{trans}$ = 0.754
    - What are the units?
  - $\sigma_g$ = exp(0.754) = 2.13
    - What are the units?

 22

## Values for $\bar{d}_{p,g}$ and $\sigma_g$ can be based on mass/area/or number of particles

- **Analyze graphically using a log-probability plot**
  - Particle diameter is plotted on the log-scale *abscissa*
  - Cumulative percent mass/area/or number of particles less than the specified diameter is plotted along the *ordinate*
- If points lie on a straight line, then the distribution is log-normal
- The geometric mass/area/or number mean diameter is the diameter with 50% by mass/area/or number of particles less than the specified diameter

2019/10/9  23

## Log-Probability Graph

Data from a cascade impactor

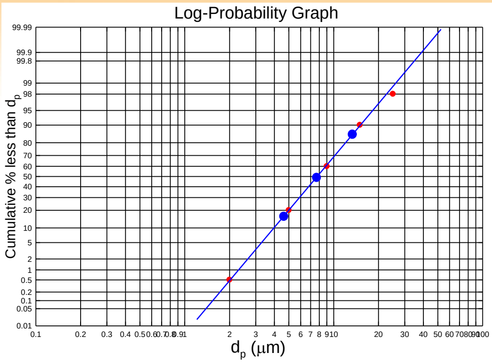
| Size range (μm) | 0-2 | 2-5 | 5-9 | 9-15 | 15-25 | >25 |
|---|---|---|---|---|---|---|
| Mass (mg) | 4.5 | 179.5 | 368 | 276 | 73.5 | 18.5 |

| Size range (μm) | Mass fraction ($m_i$) | Cumulative percent |
|---|---|---|
| 0-2 | 0.0049 | 0.5 |
| 2-5 | 0.195 | 20.0 |
| 5-9 | 0.4 | 60.0 |
| 9-15 | 0.3 | 90.0 |
| 15-25 | 0.08 | 98.0 |
| >25 | 0.02 | 100 |

**Q: Is this a log-normal distribution? If so, what's its $d_{p,g}$? $\sigma_g$?**

Get graph paper from: http://www.reliasoft.com/pubs/paper_lognormal.pdf

 24




 25

$\bar{d}_{p,g} = 50^{th}$ percentile $dp = d_{50\%}$

$\sigma_g = avg\left(\frac{d_{84.1}}{d_{50}}, \frac{d_{50}}{d_{15.9}}\right)$

Again…graph paper:
http://www.reliasoft.com/pubs/paper_lognormal.pdf

 26