Example: Analysis of Variance (One-way ANOVA)

Thirty-two tires were endurance tested. The following table shows results in months before failure.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Observations (n = 8, here)</th>
<th>Totals</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Michelin</td>
<td>49 42 48 43 50 49 50 45</td>
<td>376</td>
<td>47.00</td>
</tr>
<tr>
<td>Goodyear</td>
<td>36 36 43 34 31 36 33 30</td>
<td>279</td>
<td>34.88</td>
</tr>
<tr>
<td>Firestone</td>
<td>22 39 36 37 31 23 29 28</td>
<td>245</td>
<td>30.63</td>
</tr>
<tr>
<td>BF Goodrich</td>
<td>29 29 31 30 34 30 29 28</td>
<td>240</td>
<td>30.00</td>
</tr>
</tbody>
</table>

The general representation of the table is

<table>
<thead>
<tr>
<th>Brand/Manufacturer</th>
<th>Observations</th>
<th>Totals</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Michelin</td>
<td>y11 y12 y13 y14 y15 y16 y17 y18</td>
<td>y1. y1..avg</td>
<td></td>
</tr>
<tr>
<td>Goodyear</td>
<td>y21 y22 y23 y24 y25 y26 y27 y28</td>
<td>y2. y2..avg</td>
<td></td>
</tr>
<tr>
<td>Firestone</td>
<td>y31 y32 y33 y34 y35 y36 y37 y38</td>
<td>y3. y3..avg</td>
<td></td>
</tr>
<tr>
<td>BF Goodrich</td>
<td>y41 y42 y43 y44 y45 y46 y47 y48</td>
<td>y4. y4..avg</td>
<td></td>
</tr>
</tbody>
</table>

Now the goal is to determine if there are differences in the mean lifetimes…

1. Calculate sums of squares: SS_{total}, SS_{treat}, SS_{E}

   These equations were, in fact, CORRECT as presented in class...

   \[
   SS_{total} = \left( \sum_{i=1}^{k} \sum_{j=1}^{n} y_{ij}^2 \right) - \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{..})^2}{N} = \frac{1140^2}{32} = 1983.5
   \]

   \[
   SS_{treat} = \left( \sum_{i=1}^{k} \frac{\sum_{j=1}^{n} y_{ij}^2}{n} \right) - \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} (Y_{i.} - \bar{y}_{..})^2}{N} = \frac{376^2 + 279^2 + 245^2 + 240^2}{8} - \frac{1140^2}{32} = 1492.75
   \]

   \[
   SS_{E} = SS_{total} - SS_{treat} = 1938.5 - 1492.75 = 490.75
   \]

2. Calculate the degrees of freedom: dF_{total}, dF_{treat}, dF_{E}

   These, as presented in class, were INCORRECT (sorry for confusion)...

   \[
   dF_{total} = kn - 1 = N - 1 = 31
   \]

   \[
   dF_{treat} = k - 1 = 3
   \]

   \[
   dF_{E} = k(n - 1) = N - k = 28
   \]
3. Calculate **mean squares**: MS\(_{\text{treat}}\), MS\(_{E}\)

   *In general, MS = SS/dF*

   \[
   \text{MS}_{\text{treat}} = \frac{SS_{\text{treat}}}{dF_{\text{treat}}} = 1492.75/3 = 497.58 \\
   \text{MS}_{E} = \frac{SS_{E}}{dF_{E}} = 490.75/28 = 17.53
   \]

4. Calculate \( F = \frac{\text{MS}_{\text{treat}}}{\text{MS}_{E}} \)

   \[ F = \frac{497.58}{17.53} = 28.39 \]

5. Build **ANOVA table** and determine significance

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>1492.75</td>
<td>3</td>
<td>497.5833</td>
<td>28.3988</td>
<td>1.22E-08</td>
<td>2.946685</td>
</tr>
<tr>
<td>Within Groups</td>
<td>490.75</td>
<td>28</td>
<td>17.52679</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1983.5</td>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   Two approaches:
   1. fixed \( \alpha \)-level \( \rightarrow \) compare \( F_{\text{calc}} \) \( (28.4) \) to \( F_{\text{crit}} = F_{\alpha, a-1, a(n-1)} = 2.95 \)
   2. p-value approach \( \rightarrow \) is tabulated p-value “large” or “small”?

6. **Conclusion**: There is a difference in tire lifetimes…
   - since \( F_{\text{calc}} \) is more extreme than \( F_{\text{crit}} \) at \( \alpha = 0.05 \) (fixed \( \alpha \) approach);
   - since p-value is so small (p-value approach)